Using Dirac Notation to Analyze Single Particle Interference

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The schematic diagram below shows a Mach-Zehnder interferometer for photons. When the experiment is run so that there is only one photon in the apparatus at any time, the photon is always detected at D_2 and never at D_1.(1,2,3)

The quantum mechanical analysis of this striking phenomenon is outlined below. The photon leaves the source, S, and whether it takes the upper or lower path it interacts with a beam splitter, a mirror, and another beam splitter before reaching the detectors. At the beam splitters there is a 50% chance that the photon will be transmitted and a 50% chance that it will be reflected.

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                 Upper Path

            \ BS              \
  (S)- -> - -\- - - - T - - - -\ M
              |\               |\
              |                |
              |                |
              R                |
              |                |
              |                |
             \|            BS \|
           M \- - - - - - - - -\- - - -> D2
               \               |\              BS = Beam splitter (50/5
                 Lower Path    |               D  = Detector
                               |               M  = Mirror
                               v               S  = Source
                               D1
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After the first beam splitter the photon is in an even linear superpositio of being transmitted and reflected. Reflection involves a 90º ($\pi/2$) phase change which is represented by $\exp(i\pi/2) = i$, where $i = (-1)^{1/2}$. (See the appendix for a simple justification of the 90º phase difference between transmission and reflection.) Thus the state after the first beam is given by equation (1).

(1) $$|\psi\rangle = [|T\rangle + i|R\rangle]/2^{1/2}$$

Now $|T\rangle$ and $|R\rangle$ will be written in terms of $|D_1\rangle$ and $|D_2\rangle$ the states they evolve to at detection. $|T\rangle$ reaches $|D_1\rangle$ by transmission and $|D_2\rangle$ by relection.

(2) $$|T\rangle = [|D_1\rangle + i|D_2\rangle]/2^{1/2}$$

$|R\rangle$ reaches $|D_1\rangle$ by reflection and $|D_2\rangle$ by transmission.

(3) $$|R\rangle = [i|D_1\rangle + |D_2\rangle]/2^{1/2}$$

Equations (2) and (3) are substituted into equation (1).

(4) $|\psi\rangle = [|D_1\rangle + i|D_2\rangle + i^2|D_1\rangle + i|D_2\rangle]/2$

It is clear ($i^2 = -1$) that the first and third terms cancel (the amplitudes are 180^o out of phase), so that we end up with a final state given by equation 5.

(5) $|\psi\rangle = i|D_2\rangle$

The probability of an event is the square of the absolute magnitude of the probability amplitude.

(6) $P(D_2) = |i|^2 = 1$

Thus this analysis is in agreement with the experimental outcome that no photons are ever detected at D_1.

Appendix:

Suppose there is no phase difference between transmission and reflection. Then equations (1), (2), and (3) become

(1') $|\psi\rangle = [|T\rangle + |R\rangle]/2^{1/2}$

(2') $|T\rangle = [|D_1\rangle + |D_2\rangle]/2^{1/2}$

(3') $|R\rangle = [|D_1\rangle + |D_2\rangle]/2^{1/2}$

Substitution of equations (2') and (3') into equation (1') yields

(4') $|\psi\rangle = |D_1\rangle + |D_2\rangle$

Thus, the detection probabilities at the two detectors are:

(5') $P(D_1) = 1$ and $P(D_2) = 1$

This result violates the principle of conservation of energy because the original photon has a probability of 1 of being detected at D_1 and also a probability of 1 of being detected at D_2. In other words, the number of photons has doubled. Thus, there must be a phase difference between tra and reflection, and a 90^o phase difference, as shown above, conserves ener

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