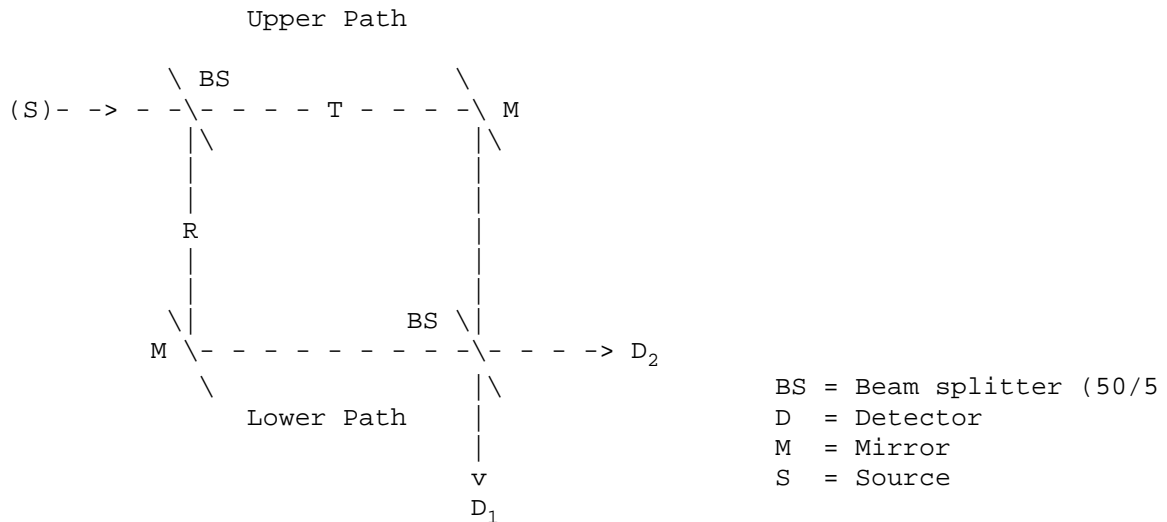


Using Dirac Notation to Analyze Single Particle Interference

Frank Rioux
Department of Chemistry
Saint John's University
College of Saint Benedict

The schematic diagram below shows a Mach-Zehnder interferometer for photons. When the experiment is run so that there is only one photon in the apparatus at any time, the photon is always detected at D_2 and never at D_1 . (1,2,3)

The quantum mechanical analysis of this striking phenomenon is outlined below. The photon leaves the source, S , and whether it takes the upper or lower path it interacts with a beam splitter, a mirror, and another beam splitter before reaching the detectors. At the beam splitters there is a 50% chance that the photon will be transmitted and a 50% chance that it will be reflected.



After the first beam splitter the photon is in an even linear superposition of being transmitted and reflected. Reflection involves a 90° ($\pi/2$) phase change which is represented by $\exp(i\pi/2) = i$, where $i = (-1)^{1/2}$. (See the appendix for a simple justification of the 90° phase difference between transmission and reflection.) Thus the state after the first beam is given by equation (1).

$$(1) \quad |\psi\rangle = [|T\rangle + i|R\rangle] / 2^{1/2}$$

Now $|T\rangle$ and $|R\rangle$ will be written in terms of $|D_1\rangle$ and $|D_2\rangle$ the states they evolve to at detection. $|T\rangle$ reaches $|D_1\rangle$ by transmission and $|D_2\rangle$ by reflection.

$$(2) \quad |T\rangle = [|D_1\rangle + i|D_2\rangle] / 2^{1/2}$$

$|R\rangle$ reaches $|D_1\rangle$ by reflection and $|D_2\rangle$ by transmission.

$$(3) \quad |R\rangle = [i|D_1\rangle + |D_2\rangle] / 2^{1/2}$$

Equations (2) and (3) are substituted into equation (1).

$$(4) \quad |\psi\rangle = [|D_1\rangle + i|D_2\rangle + i^2|D_1\rangle + i|D_2\rangle]/2$$

It is clear ($i^2 = -1$) that the first and third terms cancel (the amplitudes are 180° out of phase), so that we end up with a final state given by equation 5.

$$(5) \quad |\psi\rangle = i|D_2\rangle$$

The probability of an event is the square of the absolute magnitude of the probability amplitude.

$$(6) \quad P(D_2) = |i|^2 = 1$$

Thus this analysis is in agreement with the experimental outcome that no photons are ever detected at D_1 .

Appendix:

Suppose there is no phase difference between transmission and reflection. Then equations (1), (2), and (3) become

$$(1') \quad |\psi\rangle = [|T\rangle + |R\rangle]/2^{1/2}$$

$$(2') \quad |T\rangle = [|D_1\rangle + |D_2\rangle]/2^{1/2}$$

$$(3') \quad |R\rangle = [|D_1\rangle + |D_2\rangle]/2^{1/2}$$

Substitution of equations (2') and (3') into equation (1') yields

$$(4') \quad |\psi\rangle = |D_1\rangle + |D_2\rangle$$

Thus, the detection probabilities at the two detectors are:

$$(5') \quad P(D_1) = 1 \quad \text{and} \quad P(D_2) = 1$$

This result violates the principle of conservation of energy because the original photon has a probability of 1 of being detected at D_1 and also a probability of 1 of being detected at D_2 . In other words, the number of photons has doubled. Thus, there must be a phase difference between transmission and reflection, and a 90° phase difference, as shown above, conserves energy.

References:

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