Time-dependent Wigner Function for Harmonic Oscillator Transitions

Frank Rioux

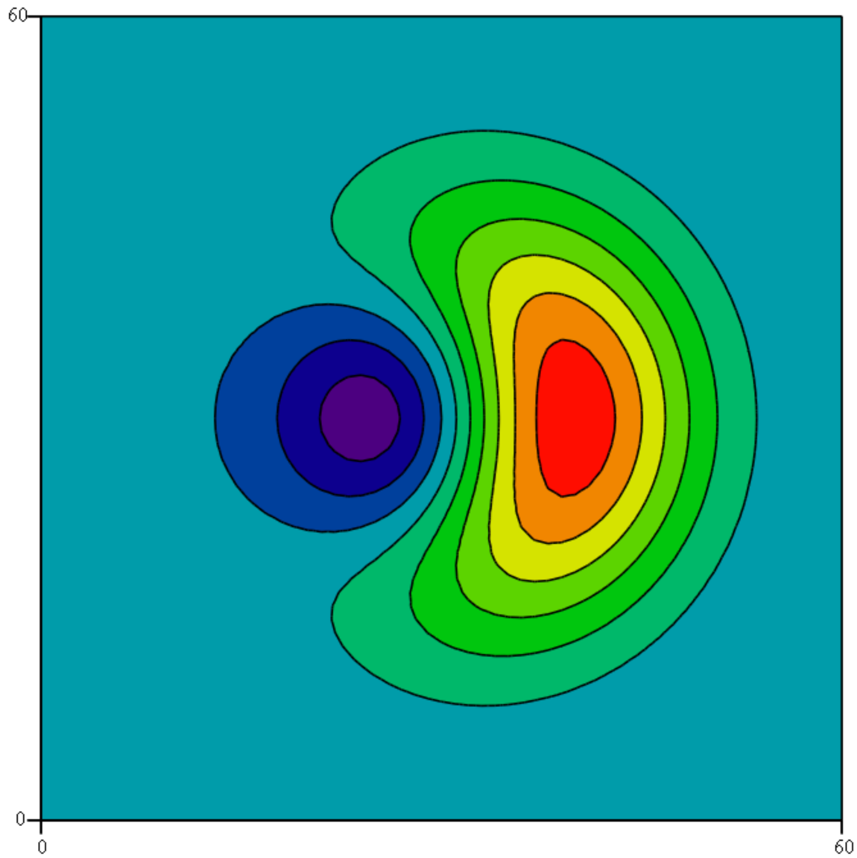
Initial state: $m := 0$ $E_m := m + \frac{1}{2}$ Final state: $n := 1$ $E_n := n + \frac{1}{2}$ $t := FRAME$

Define Wigner distribution function for a linear superposition of the initial and final harmonic oscillator state.

$$W(x,p) := \frac{1}{\pi^{\frac{3}{2}}} \cdot \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2^n \cdot n! \cdot \sqrt{\pi}}} \cdot Her\left(n, x + \frac{s}{2}\right) \cdot \exp\left[-\frac{\left(x + \frac{s}{2}\right)^2}{2}\right] \cdot \exp\left(i \cdot E_n \cdot t\right) \ldots + \frac{1}{\sqrt{2^m \cdot m! \cdot \sqrt{\pi}}} \cdot Her\left(m, x + \frac{s}{2}\right) \cdot \exp\left[-\frac{\left(x + \frac{s}{2}\right)^2}{2}\right] \cdot \exp\left(i \cdot E_m \cdot t\right) \right] \cdot \exp(i \cdot s \cdot p) \cdot \left[\frac{1}{\sqrt{2^n \cdot n! \cdot \sqrt{\pi}}} \cdot Her\left(n, x - \frac{s}{2}\right) \cdot \exp\left[-\frac{\left(x - \frac{s}{2}\right)^2}{2}\right] \cdot \exp\left(-i \cdot E_n \cdot t\right) \ldots + \frac{1}{\sqrt{2^m \cdot m! \cdot \sqrt{\pi}}} \cdot Her\left(m, x - \frac{s}{2}\right) \cdot \exp\left[-\frac{\left(x - \frac{s}{2}\right)^2}{2}\right] \cdot \exp\left(-i \cdot E_m \cdot t\right) \right] ds$$

Display Wigner distribution:

$N := 60$ $i := 0..N$ $x_i := -2.5 + \frac{5 \cdot i}{N}$ $j := 0..N$ $p_j := -2.5 + \frac{5 \cdot j}{N}$ $Wigner_{i,j} := W\left(x_i, p_j\right)$



Wigner