The Wigner Function for the Single Slit Diffraction Problem

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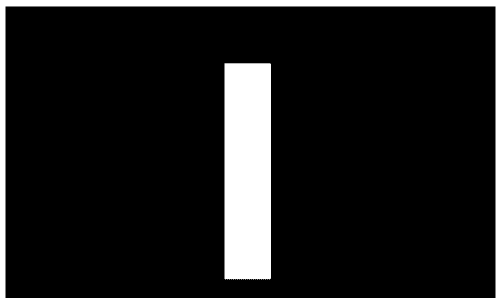
The quantum mechanical interpretation of the single-slit experiment is that position is measured at the slit screen and momentum is measured at the detection screen. Position and momentum are conjugate observables connected by a Fourier transform and governed by the uncertainty principle. Knowing the slit screen geometry makes it possible to calculate the momentum distribution at the detection screen.

The slit-screen geometry and therefore the coordinate wavefunction is calculate as follows.

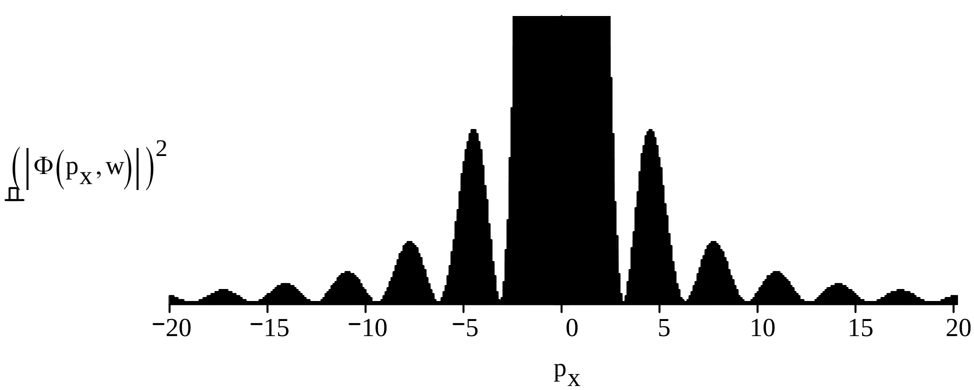
Slit width: $w := 2$ Coordinate-space wave function: $\Psi(x, w) := \text{if}\left[\left(x \geq -\frac{w}{2}\right)\cdot\left(x \leq \frac{w}{2}\right), 1, 0\right]$

$$x := \frac{-w}{2}, \frac{-w}{2} + .005 .. \frac{w}{2}$$



A Fourier transform of the coordinate-space wave function yields the momentum wave function and the momentum distribution function, which is the diffraction pattern.

$$\Phi\left(p_x, w\right) := \frac{1}{\sqrt{2\cdot\pi\cdot w}}\cdot\int_{-\frac{w}{2}}^{\frac{w}{2}} \exp\left(-i\cdot p_x\cdot x\right) dx \text{ simplify } \rightarrow 2^{\frac{1}{2}}\cdot\frac{\sin\left(\frac{1}{2}\cdot w\cdot p_x\right)}{\pi^{\frac{1}{2}}\cdot w^{\frac{1}{2}}\cdot p_x}$$

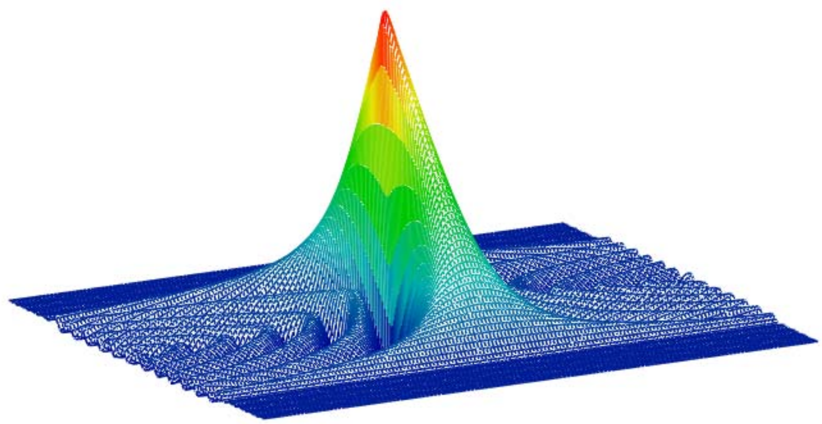


The Wigner function for the single-slit screen geometry is generated using the momentum wave function. (Fifty is effectively infinity and is therefore as the limits of integration.)

$$W(x, p) := \frac{1}{2\cdot\pi}\cdot\int_{-50}^{50} \overline{\Phi\left(p + \frac{s}{2}, w\right)}\cdot\exp(-i\cdot s\cdot x)\cdot\Phi\left(p - \frac{s}{2}, w\right) ds$$

The single-slit Wigner function is displayed graphically.

$N := 150 \quad i := 0..N \quad x_i := -1.5 + \frac{3 \cdot i}{N} \quad j := 0..N \quad p_j := -20 + \frac{40 \cdot j}{N} \quad Wigner_{i,j} := W(x_i, p_j)$



Wigner