The Wigner Distribution Function for the Harmonic Oscillator

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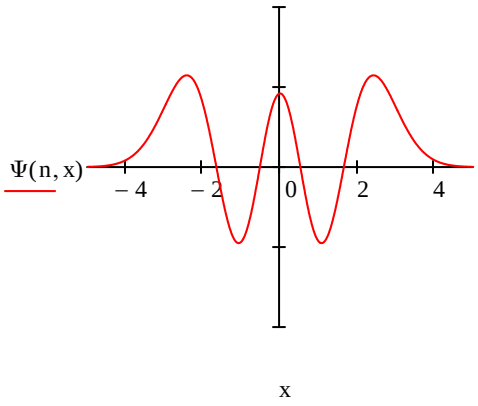
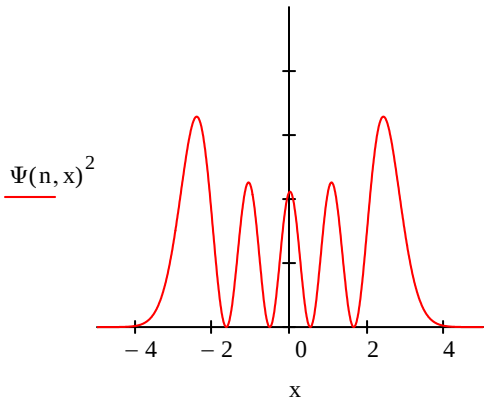
Given the quantum number this Mathcad file calculates the Wigner distribution function for the specified harmonic oscillator eigenstate using the coordinate wave function.

Quantum number: $n := 4$

Harmonic oscillator coordinate eigenstate: $\Psi(n,x) := \frac{1}{\sqrt{2^n \cdot n! \cdot \sqrt{\pi}}} \cdot Her(n,x) \cdot \exp\left(-\frac{x^2}{2}\right)$

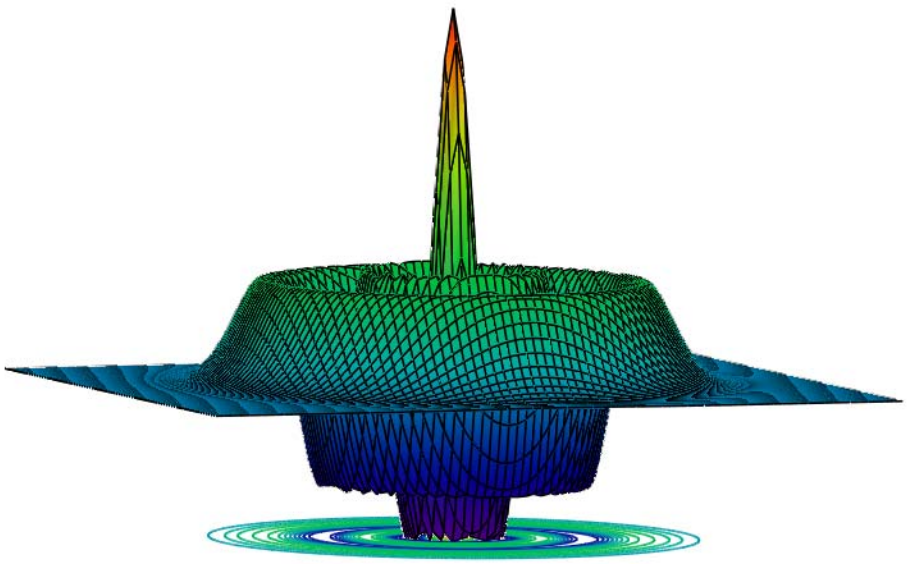
Display coordinate wave function and distribution function:





Calculate Wigner distribution: $W(n,x,p) := \frac{1}{\pi^{\frac{3}{2}}} \cdot \int_{-\infty}^{\infty} \Psi\left(n, x + \frac{s}{2}\right) \cdot \exp(i \cdot s \cdot p) \cdot \Psi\left(n, x - \frac{s}{2}\right) ds$

Display Wigner distribution: $N := 80$ $i := 0 .. N$ $x_i := -4 + \frac{8 \cdot i}{N}$ $j := 0 .. N$ $p_j := -5 + \frac{10 \cdot j}{N}$ $Wigner_{i,j} := W\left(n, x_i, p_j\right)$



Calculate the momentum distribution function using the Wigner function: $\rho(p) := \int_{-\infty}^{\infty} W(n, x, p)\, dx \quad p := -5, -4.95 .. 5$