The Wigner Function for the 2p State of the 1D Hydrogen Atom

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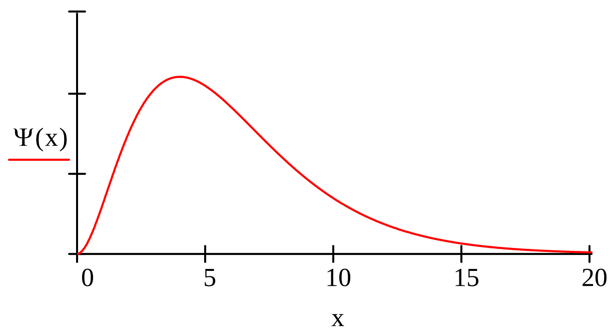
This tutorial presents three pictures of the 2p state of the one-dimensional hydrogen atom using its position, momentum and phase-space representations.

The energy operator for the one-dimensional hydrogen atom in atomic units is:

$$\frac{-1}{2}\cdot\frac{d^2}{dx^2}\blacksquare + \frac{L\cdot(L+1)}{2\cdot x^2}\cdot\blacksquare - \frac{1}{x}\cdot\blacksquare$$

The 2p wave function is:

$$\Psi(x) := \frac{1}{\sqrt{24}}\cdot x^2\cdot \exp\left(-\frac{x}{2}\right) \qquad \int_0^{\infty}\Psi(x)^2\,dx = 1$$

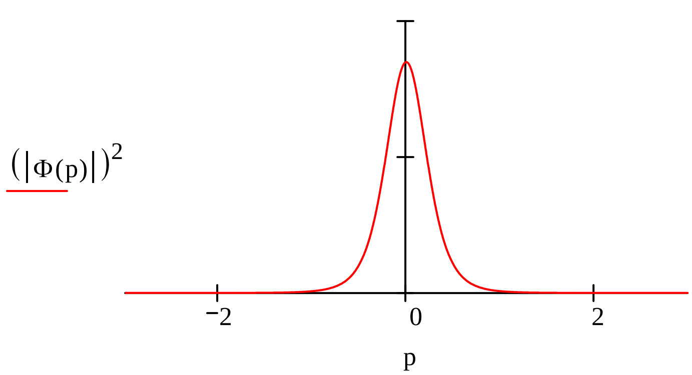


The 2p state energy is -0.125 E_h.

$$\frac{-1}{2}\cdot\frac{d^2}{dx^2}\Psi(x) + \frac{1}{x^2}\cdot\Psi(x) - \frac{1}{x}\cdot\Psi(x) = E\cdot\Psi(x) \text{ solve}, E \rightarrow \frac{-1}{8} = -0.125$$

The momentum wave function is generated by the following Fourier transform of the coordinate space wave function.

$$\Phi(p) := \frac{1}{\sqrt{2\cdot\pi}}\cdot\int_0^{\infty}\exp(-i\cdot p\cdot x)\cdot\Psi(x)\,dx \rightarrow \frac{2}{3}\cdot 2^{\frac{1}{2}}\cdot\frac{6^{\frac{1}{2}}}{(2\cdot i\cdot p+1)^3\cdot\pi^{\frac{1}{2}}}$$

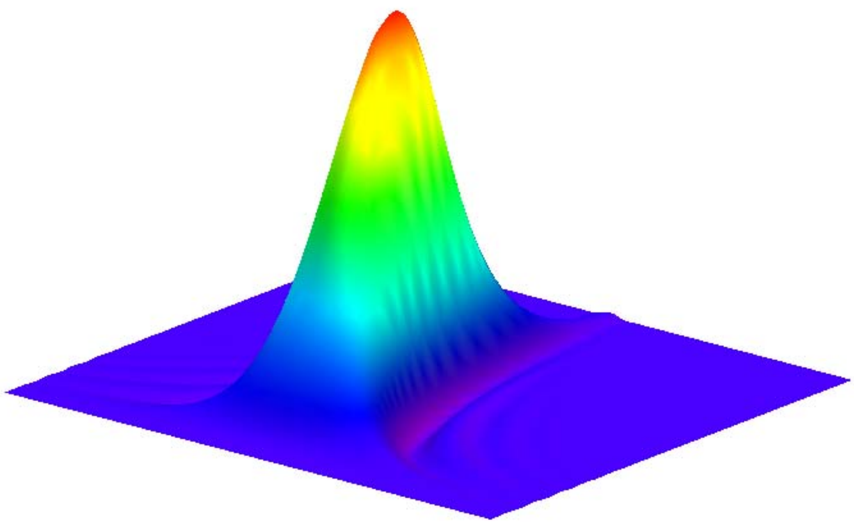


The Wigner function (phase-space representation) for the 2p state is generated using the momentum wave function.

$$W(x, p) := \frac{1}{2 \cdot \pi} \cdot \int_{-\infty}^{\infty} \overline{\Phi\left(p + \frac{s}{2}\right)} \cdot \exp(-i \cdot s \cdot x) \cdot \Phi\left(p - \frac{s}{2}\right) ds$$

The Wigner distribution is displayed graphically.

$N := 100$ $\quad i := 0 .. N$ $\quad x_i := \frac{15 \cdot i}{N}$ $\quad j := 0 .. N$ $\quad p_j := -3 + \frac{6 \cdot j}{N}$ $\quad \text{Wigner}_{i, j} := W(x_i, p_j)$



Wigner