

The Temporal Double-Slit Experiment

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In the familiar spatial double-slit experiment illumination of the slit screen leads to a "Schrodinger cat state" in which photons or other quons are in a linear superposition of being present at both slits simultaneously.

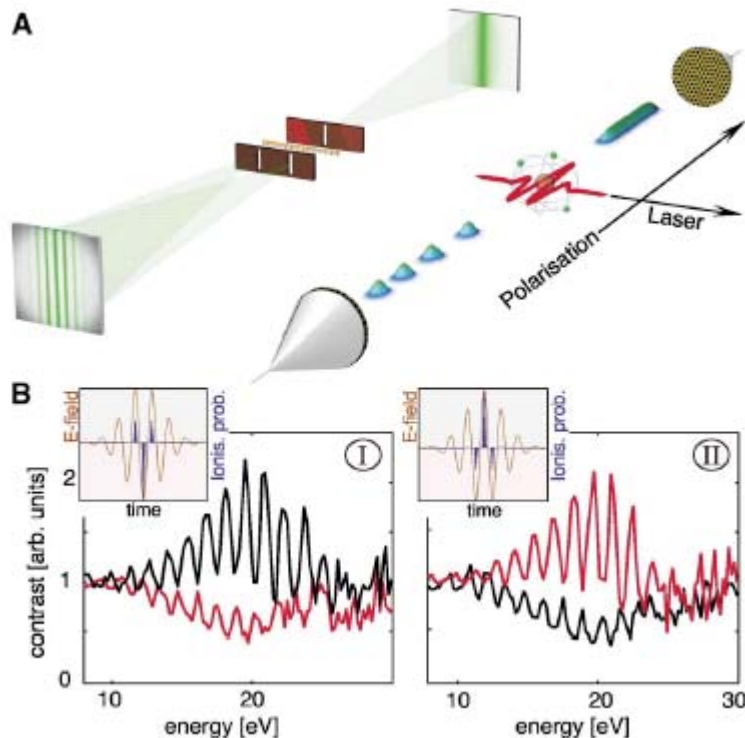
$$|\Psi\rangle = \frac{1}{\sqrt{2}}[|x_1\rangle + |x_2\rangle]$$

This spatial superposition leads to interference fringes at the detection screen. See for example the previous tutorial (<http://www.users.csbsju.edu/~frioux/two-slit/another2slit.pdf>).

Recently **Physics World** reported a temporal version of the double-slit experiment performed by an international team led by Gerhard Paulus (<http://www.physicsweb.org/articles/news/9/3/1/1>). In this remarkable experiment a 5-femtosecond laser pulse, which consists of two maxima and one minimum in the electric field, is used to ionize argon atoms.



As shown in the figure below, the maxima and minimum accelerate the ionized photons in opposite directions.



According to quantum mechanics the electrons traveling to the right-hand detector are represented by a linear superposition of being ionized at two different times, say t_1 and t_2 .

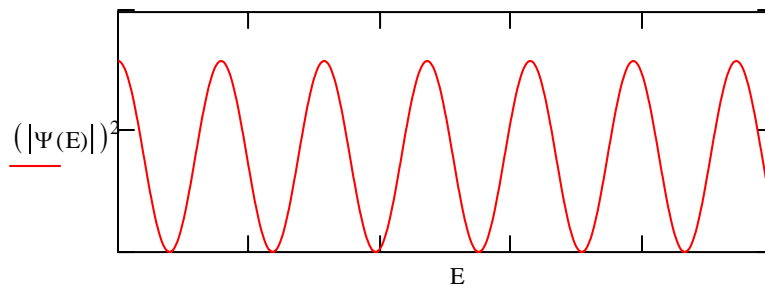
$$|\Psi\rangle = \frac{1}{\sqrt{2}}[|t_1\rangle + |t_2\rangle]$$

At the detector Paulus et al. measure the kinetic energy of the electrons and observe the interference fringes shown in the figure above. The fringes can be understood by projecting this linear superposition onto the energy representation.

$$\langle E | \Psi \rangle = \frac{1}{\sqrt{2}} [\langle E | t_1 \rangle + \langle E | t_2 \rangle] = \frac{1}{\sqrt{2}} \left[\exp\left(-\frac{iEt_1}{\hbar}\right) + \exp\left(-\frac{iEt_2}{\hbar}\right) \right]$$

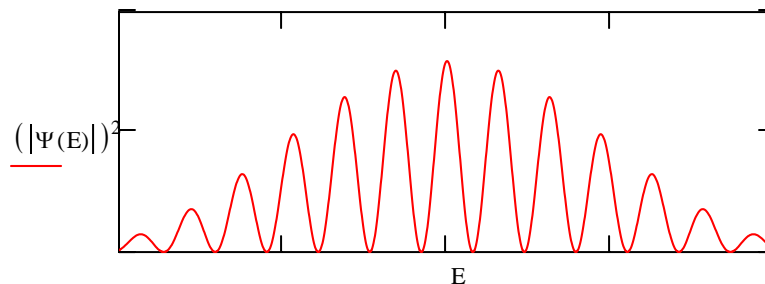
Clearly, there are two temporal "paths" for each observed energy value and they interfere with each other constructively and destructively. Plotting $|\langle E | \Psi \rangle|^2$ shows that this is the origin of the observed fringes.

Times of maxima: $t_1 := 1$ $t_2 := 5$ $\Psi(E) := \frac{\frac{1}{\sqrt{2 \cdot \pi}} \cdot \exp(-i \cdot E \cdot t_1) + \frac{1}{\sqrt{2 \cdot \pi}} \cdot \exp(-i \cdot E \cdot t_2)}{\sqrt{2}}$



This calculation shows no attenuation in the fringes at the extremes of energy because there is no uncertainty in the time of the pulse maxima. A calculation which introduces an uncertainty of δ in the duration of each maximum yields a more realistic picture of the interference fringes.

$\delta := .5$ $\Psi(E) := \frac{\int_{t_1 - \frac{\delta}{2}}^{t_1 + \frac{\delta}{2}} \frac{1}{\sqrt{2 \cdot \pi}} \cdot \exp(-i \cdot E \cdot t) dt + \int_{t_2 - \frac{\delta}{2}}^{t_2 + \frac{\delta}{2}} \frac{1}{\sqrt{2 \cdot \pi}} \cdot \exp(-i \cdot E \cdot t) dt}{\sqrt{2}}$



I wish to gratefully acknowledge experimental details generously supplied by Professor Gerhard Paulus of Texas A&M University.

Relevant references:

G. G. Paulus, F. Lindner, H. Walther, A. Baltuska, E. Goulielmakis, M. Lezius, F. Krausz, "Measurement of the phase of few-cycle laser pulses," *Physical Review Letters* **91**, 253004 (2003)

F. Lindner, G. G. Paulus, H. Walther, A. Baltuska, E. Goulielmakis, M. Lezius, F. Krausz, "Gouy phase shift for few-cycle laser pulses," *Physical Review Letters* **92**, 113001 (2004)