Which Way Did It Go? - The Quantum Eraser

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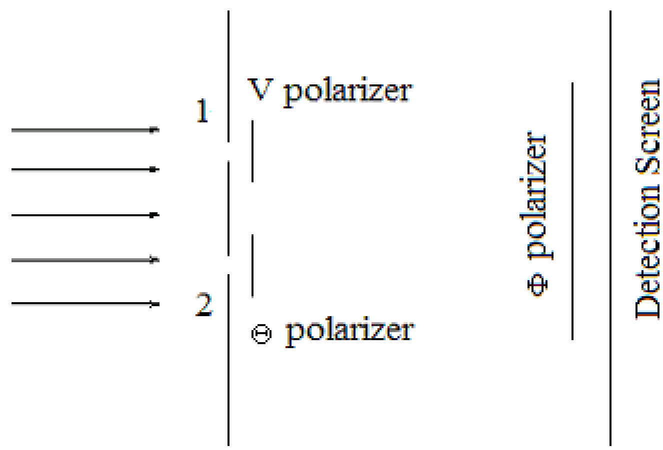
Paul Kwiat and Rachel Hillmer, an undergraduate research assistant, published "A Do-It-Yourself Quantum Eraser" based on the double-slit experiment in the May 2007 issue of *Scientific American*. The purpose of this tutorial is to show the quantum math behind the laser demonstrations illustrated in this article.

Hillmer and Kwiat created the double-slit effect by illuminating a thin wire with a laser beam. They carried out a number of demonstrations with laser light and polarizing films using an experimental set up that effectively is as shown schematically below.



Assuming (initially) infinitesimally thin slits, the photon wave function at the slit screen is an entangled superposition of being at slit 1 with vertical polarization and slit 2 with polarization at an angle θ relative to the vertical. This entanglement provides which-way information if θ is not equal to 0 and, therefore, has an important effect on the diffraction pattern.

$$\left|\Psi\right\rangle = \frac{1}{\sqrt{2}}\left[\left|x_1\right\rangle\left|v\right\rangle + \left|x_2\right\rangle\left|\theta\right\rangle\right]$$

This state is projected onto ϕ and **p** because a ϕ-oriented polarizer (eraser) precedes the detection screen and because a diffraction pattern is actually the momentum distribution of the scattered photons. In other words, position is measured at the slit screen and momentum is measured at the detection screen.

$$\left\langle p\phi\middle|\Psi\right\rangle = \frac{1}{\sqrt{2}}\left[\left\langle p\middle|x_1\right\rangle\left\langle\phi\middle|v\right\rangle + \left\langle p\middle|x_2\right\rangle\left\langle\phi\middle|\theta\right\rangle\right]$$

The polarization brackets (amplitudes) are easily shown to be the trigonometric functions shown below.

$$\left\langle p\phi\middle|\Psi\right\rangle = \frac{1}{\sqrt{2}}\left[\left\langle p\middle|x_1\right\rangle\cos(\phi) + \left\langle p\middle|x_2\right\rangle\cos(\theta-\phi)\right]$$

The position-momentum brackets are the position eigenstates in the momentum representation and are given by,

$$\langle p | x \rangle = \frac{1}{\sqrt{2\pi\hbar}} \exp\left(-\frac{ipx}{\hbar}\right)$$

This allows us to write,

$$\langle p\phi | \Psi \rangle = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2\pi\hbar}} \exp\left(-\frac{ipx_1}{\hbar}\right)\cos(\phi) + \frac{1}{\sqrt{2\pi\hbar}} \exp\left(-\frac{ipx_2}{\hbar}\right)\cos(\theta - \phi)\right]$$

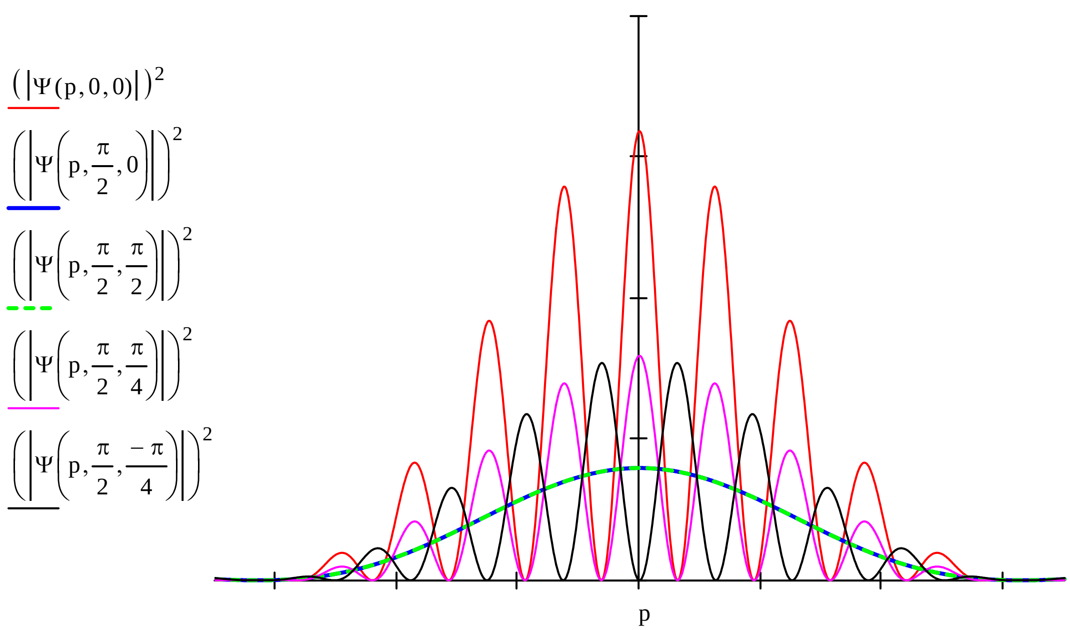
Working in atomic units (h = 2π) and now assuming slits of finite width this expression becomes,

Slit positions: $x_1 := 1$ $x_2 := 2$ Slit width: $\delta := .2$

$$\Psi(p,\theta,\phi) := \frac{\displaystyle\int_{x_1-\frac{\delta}{2}}^{x_1+\frac{\delta}{2}} \frac{1}{\sqrt{2\cdot\pi}}\cdot\exp(-i\cdot p\cdot x)\cdot\frac{1}{\sqrt{\delta}}\,dx\cdot\cos(\phi) + \int_{x_2-\frac{\delta}{2}}^{x_2+\frac{\delta}{2}} \frac{1}{\sqrt{2\cdot\pi}}\cdot\exp(-i\cdot p\cdot x)\cdot\frac{1}{\sqrt{\delta}}\,dx\cdot\cos(\theta-\phi)}{\sqrt{2}}$$

The square of the absolute magnitude of this function yields a representation of the diffraction pattern as a histogram of photon arrivals on the detection screen. The results shown in the figure will be discussed below.

Histogram of Detected Photons



Discussion of Results

The polarizer at slit 1 is always oriented vertically so only the orientations (θ and ϕ) of the other polarizers need to be specified.

[θ = 0; ϕ = 0] The photons emerging from the slits are vertically polarized and encounter a vertical polarizer before the detection screen. This is the reference experiment and yields the traditional diffraction pattern, as shown by the plot of $(|\Psi(p,0,0)|)^2$. There is no which-way information in this experiment and 100% of the photons emerging from the vertically polarized slit screen reach the detection screen.

$$\int_{-\infty}^{\infty} (|\Psi(p,0,0)|)^2 \, dp \text{ float}, 3 \rightarrow 1.00$$

[θ = π/2, ϕ = 0] and [θ = π/2, ϕ = π/2] The crossed polarizers at the slit screen provide which-way information and the interference fringes disappear if the third polarizer is vertically or horizontally oriented. This is shown by the plots of $\left(\left|\Psi\left(p,\frac{\pi}{2},0\right)\right|\right)^2$ and $\left(\left|\Psi\left(p,\frac{\pi}{2},\frac{\pi}{2}\right)\right|\right)^2$. Furthermore, relative to the reference experiment, 50% of the photons reach the detection screen.

$$\int_{-\infty}^{\infty} \left(\left|\Psi\left(p,\frac{\pi}{2},0\right)\right|\right)^2 dp \text{ float}, 3 \rightarrow .500 \qquad \int_{-\infty}^{\infty} \left(\left|\Psi\left(p,\frac{\pi}{2},\frac{\pi}{2}\right)\right|\right)^2 dp \text{ float}, 3 \rightarrow .500$$

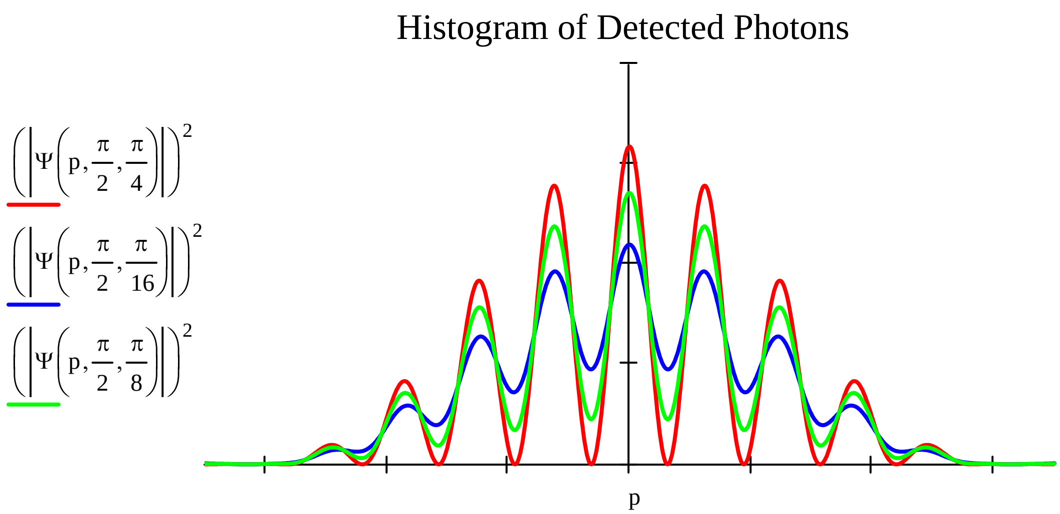
In the absence of the third polarizer, there is also no diffraction pattern but 100% of the photons reach the detection screen.

[θ = π/2, ϕ = π/4] and [θ = π/2, ϕ = -π/4] The which-way information provided by the crossed polarizers at the slit screen is erased by diagonally and anti-diagonally oriented polarizers in front of the detection screen. This is shown by the plots of $\left(\left|\Psi\left(p,\frac{\pi}{2},\frac{\pi}{4}\right)\right|\right)^2$ and $\left(\left|\Psi\left(p,\frac{\pi}{2},-\frac{\pi}{4}\right)\right|\right)^2$. The reason the which-way information has been erased is that vertically and horizontally polarized photons emerging from slits 1 and 2 both have a 50% chance of passing the diagonally or anti-diagonally oriented third polarizer. Thus, it is impossible to determine the origin of a photon that passes the third polarizer and the interference fringes are restored. Again, for this experiment 50% of the photons reach the detection screen.

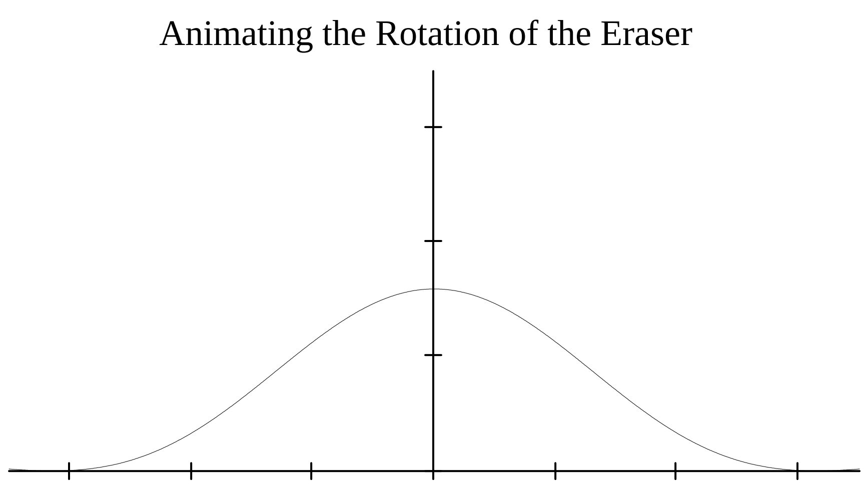
$$\int_{-\infty}^{\infty} \left(\left|\Psi\left(p,\frac{\pi}{2},\frac{\pi}{4}\right)\right|\right)^2 dp \text{ float}, 3 \rightarrow .500 \qquad \int_{-\infty}^{\infty} \left(\left|\Psi\left(p,\frac{\pi}{2},-\frac{\pi}{4}\right)\right|\right)^2 dp \text{ float}, 3 \rightarrow .500$$

The shift in the interference fringes calculated for $\left(\left|\Psi\left(p,\frac{\pi}{2},\frac{\pi}{4}\right)\right|\right)^2$ and $\left(\left|\Psi\left(p,\frac{\pi}{2},-\frac{\pi}{4}\right)\right|\right)^2$ is observed in the Kwiat/Hillmer experiment.

The visibility of the restored fringes is maximized for ϕ = +/- $\pi/4$. As the figure belows shows the visibility is reduced for other values of ϕ.



It is possible to animate the rotation of the polarizer in front of the detection screen, the eraser. From Tools select Animation and use the following setting: **From: 0 To: 120 At: 5 Frames/Sec**.



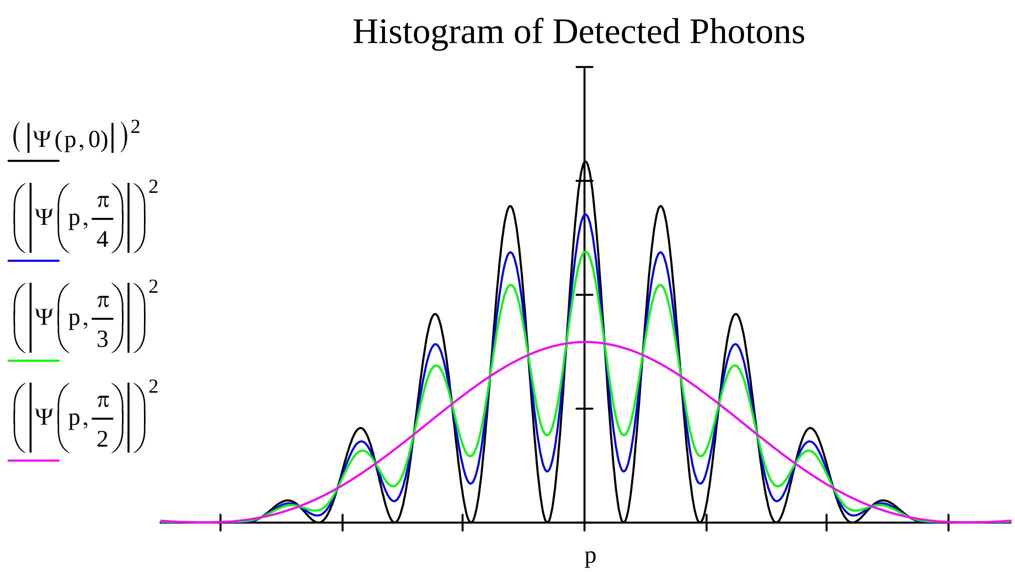
Explicit Vector Approach

In what follows an explicit vector approach to the analysis above is provided.

$$\frac{1}{\sqrt{2}}\left[\left|x_1\right\rangle\left|v\right\rangle+\left|x_2\right\rangle\left|\theta\right\rangle\right]=\frac{1}{\sqrt{2}}\left[\left|x_1\right\rangle\begin{pmatrix}1\\0\end{pmatrix}+\left|x_2\right\rangle\begin{pmatrix}\cos(\theta)\\ \sin(\theta)\end{pmatrix}\right]\xrightarrow{\left\langle p\right|}\frac{1}{\sqrt{2}}\begin{pmatrix}\left\langle p|x_1\right\rangle+\cos(\theta)\left\langle p|x_2\right\rangle\\ \sin(\theta)\left\langle p|x_2\right\rangle\end{pmatrix}$$

Assuming finite slit widths, the <p|x> amplitudes become integrals as outlined above. The appropriate Mathcad expression and its graphical display is shown below.

$$\Psi(p,\theta) := \frac{1}{2\cdot\sqrt{\pi}}\cdot\begin{pmatrix}\displaystyle\int_{x_1-\frac{\delta}{2}}^{x_1+\frac{\delta}{2}}\exp(-i\cdot p\cdot x)\cdot\frac{1}{\sqrt{\delta}}\,dx+\cos(\theta)\cdot\int_{x_2-\frac{\delta}{2}}^{x_2+\frac{\delta}{2}}\exp(-i\cdot p\cdot x)\cdot\frac{1}{\sqrt{\delta}}\,dx\\ \sin(\theta)\cdot\displaystyle\int_{x_2-\frac{\delta}{2}}^{x_2+\frac{\delta}{2}}\exp(-i\cdot p\cdot x)\cdot\frac{1}{\sqrt{\delta}}\,dx\end{pmatrix}$$



Next Ψ(p,θ) is projected onto the eraser polarizer oriented at an angle ϕ, and the probability distributions for several combinations of θ and ϕ are displayed.

$$\Psi(p,\theta,\phi)=\begin{pmatrix}\cos(\phi) & \sin(\phi)\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}\left\langle p|x_1\right\rangle+\cos(\theta)\left\langle p|x_2\right\rangle\\ \sin(\theta)\left\langle p|x_2\right\rangle\end{pmatrix}$$

$$\Psi(p,\theta,\phi) := \begin{pmatrix}\cos(\phi) & \sin(\phi)\end{pmatrix}\cdot\Psi(p,\theta)$$