# Terse Analysis of Triple-slit Diffraction with a Quantum Eraser

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Slit positions, slit width and the wavefunction at the slit screen which is a superposition of the photon being simultaneously present at all three slits.

$$x_1 := -\frac{1}{2} \qquad x_2 := 0 \qquad x_3 := \frac{1}{2} \qquad \delta := .1$$

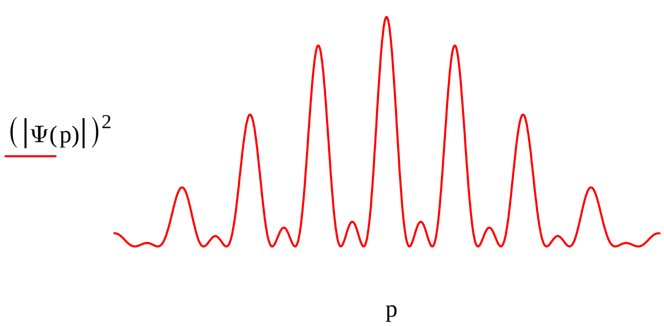
$$|\Psi\rangle = \frac{1}{\sqrt{3}}\left[|x_1\rangle + |x_2\rangle + |x_3\rangle\right]$$

Calculate the diffraction pattern by a Fourier transform of the spatial wavefunction into momentum space.

$$\langle p|\Psi\rangle = \frac{1}{\sqrt{3}}\left[\langle p|x_1\rangle + \langle p|x_2\rangle + \langle p|x_3\rangle\right]$$

$$\Psi(p) := \int_{x_1-\frac{\delta}{2}}^{x_1+\frac{\delta}{2}} \frac{1}{\sqrt{2\cdot\pi}}\cdot\exp(-i\cdot p\cdot x)\cdot\frac{1}{\sqrt{\delta}}\,dx + \int_{x_2-\frac{\delta}{2}}^{x_2+\frac{\delta}{2}} \frac{1}{\sqrt{2\cdot\pi}}\cdot\exp(-i\cdot p\cdot x)\cdot\frac{1}{\sqrt{\delta}}\,dx + \int_{x_3-\frac{\delta}{2}}^{x_3+\frac{\delta}{2}} \frac{1}{\sqrt{2\cdot\pi}}\cdot\exp(-i\cdot p\cdot x)\cdot\frac{1}{\sqrt{\delta}}\,dx$$

Display the momentum distribution function which is the diffraction pattern.



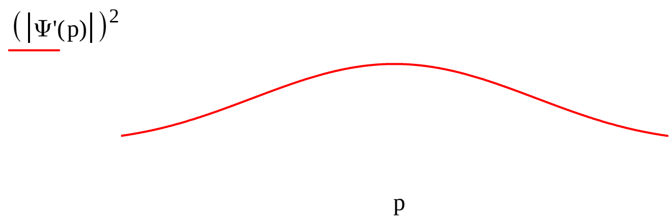

Tag the slits with orthogonal states.

$$\langle p|\Psi'\rangle = \frac{1}{\sqrt{3}}\left[\langle p|x_1\rangle|\uparrow\rangle + \langle p|x_2\rangle|\rightarrow\rangle + \langle p|x_3\rangle|\downarrow\rangle\right]$$

Recalculate the momentum distribution.

$$\Psi'(p) := \int_{x_1-\frac{\delta}{2}}^{x_1+\frac{\delta}{2}} \frac{1}{\sqrt{2\cdot\pi}}\cdot\exp(-i\cdot p\cdot x)\cdot\frac{1}{\sqrt{\delta}}\,dx\cdot\begin{pmatrix}1\\0\\0\end{pmatrix} + \int_{x_2-\frac{\delta}{2}}^{x_2+\frac{\delta}{2}} \frac{1}{\sqrt{2\cdot\pi}}\cdot\exp(-i\cdot p\cdot x)\cdot\frac{1}{\sqrt{\delta}}\,dx\cdot\begin{pmatrix}0\\1\\0\end{pmatrix} + \int_{x_3-\frac{\delta}{2}}^{x_3+\frac{\delta}{2}} \frac{1}{\sqrt{2\cdot\pi}}\cdot\exp(-i\cdot p\cdot x)\cdot\frac{1}{\sqrt{\delta}}\,dx\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}$$

Display the momentum distribution at the detection screen showing that the diffraction pattern has disappeared. The orthogonallity of the tags destroys the cross-terms in the momentum distribution, $|\Psi'(p)|^2$, which give rise to the interference effects shown in the original diffraction pattern.




Insert an "eraser" after the slit screen and before the detection screen.

$$\Psi''(p) := \frac{1}{\sqrt{3}} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}^T \cdot \Psi'(p)$$

The diffraction pattern is restored but attenuated because the so-called "eraser" filters out the orthogonal tags restoring the interference terms.