The Bohr Model of the Earth-Sun System*

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Assuming the earth executes a circular orbit of radius **r** about the sun and has a deBroglie wavelength given by **h/mv**, yields a quantum mechanical kinetic energy for the earth which is the first term in the total energy expression below. The potential energy of the earth-sun interaction is well-known and is the second term in the total energy expression.

$$E = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot Me \cdot r^2} - \frac{G \cdot Me \cdot Ms}{r}$$

Setting the first derivative of the energy with respect to **r** equal to zero, yields the allowed values of **r** in terms of the quantum number, **n**.

$$\frac{d}{dr}\left(\frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot Me \cdot r^2} - \frac{G \cdot Me \cdot Ms}{r}\right) = 0 \text{ solve}, r \rightarrow \frac{1}{4} \cdot n^2 \cdot \frac{h^2}{G \cdot Me^2 \cdot Ms \cdot \pi^2}$$

Substitution of this value of **r** in the total energy expression yields the energy of the earth-sun system as a function of the quantum number, **n**, Planck's constant, the gravitational constant, and the masses of the earth and the sun.

$$E = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot Me \cdot r^2} - \frac{G \cdot Me \cdot Ms}{r} \text{ substitute}, r = \frac{1}{4} \cdot n^2 \cdot \frac{h^2}{G \cdot Me^2 \cdot Ms \cdot \pi^2} \rightarrow E = \frac{-2}{n^2 \cdot h^2} \cdot \pi^2 \cdot Me^3 \cdot G^2 \cdot Ms^2$$

Data for the earth-sun system assuming a circular earth orbit:

Mass of the earth: $Me := 5.974 \cdot 10^{24} \cdot kg$ Mass of the sun: $Ms := 1.989 \cdot 10^{30} \cdot kg$

Earth orbit radius: $r := 1.496 \cdot 10^{11} \cdot m$ Gravitational constant: $G := 6.674 \cdot 10^{-11} \cdot \frac{N \cdot m^2}{kg^2}$

Planck's constant: $h := 6.62608 \cdot 10^{-34} \cdot J \cdot s$

Given the radius of the earth's orbit listed above, calculate the earth's quantum number.

$$r = \frac{1}{4} \cdot n^2 \cdot \frac{h^2}{G \cdot Me^2 \cdot Ms \cdot \pi^2} \;\Big|\; \begin{matrix}\text{solve}, n \\ \text{float}, 3\end{matrix} \rightarrow \begin{bmatrix} .252e75 \cdot (m \cdot kg \cdot N)^{\frac{1}{2}} \cdot \frac{m}{s \cdot J} \\ (-.252e75) \cdot (m \cdot kg \cdot N)^{\frac{1}{2}} \cdot \frac{m}{s \cdot J} \end{bmatrix} = \begin{pmatrix} 2.52 \times 10^{74} \\ -2.52 \times 10^{74} \end{pmatrix}$$

The positive root for **n** is used to calculate the energy of the earth-sun system.

$n := 2.52 \cdot 10^{74}$ $E := \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot Me \cdot r^2} - \frac{G \cdot Me \cdot Ms}{r}$ $E = -2.66 \times 10^{33} J$

According to the virial theorem the classical expression for the energy of the earth-sun system with earth orbit radius **r** is half the potential energy. Note that this gives a value which is in agreement with the Bohr model for the earth-sun system. Is this a legitimate example of the correspondence principle?

$$E := -\frac{G \cdot Me \cdot Ms}{2 \cdot r} \qquad E = -2.65 \times 10^{33}\,J$$

*Johnson and Pedersen, *Problems and Solutions in Quantum Chemistry and Physics*, pages 26-27.