

The Bohr Model of the Earth-Sun System*

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Data for the earth-sun system assuming a circular earth orbit:

Mass of the earth: $M_e := 5.974 \cdot 10^{24} \cdot \text{kg}$ Mass of the sun: $M_s := 1.989 \cdot 10^{30} \cdot \text{kg}$

Earth orbit radius: $r := 1.496 \cdot 10^{11} \cdot \text{m}$ Gravitational constant: $G := 6.674 \cdot 10^{-11} \cdot \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$

Planck's constant: $h := 6.62608 \cdot 10^{-34} \cdot \text{J} \cdot \text{s}$

Assuming the earth executes a circular orbit of radius r about the sun and has a deBroglie wavelength given by h/mv , yields a quantum mechanical kinetic energy for the earth which is the first term in the total energy expression below. The potential energy of the earth-sun interaction is well-known and is the second term in the total energy expression.

$$E = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot M_e \cdot r^2} - \frac{G \cdot M_e \cdot M_s}{r} \quad \text{where } n = 1, 2, 3, \dots$$

Setting the first derivative of the energy with respect to r equal to zero, yields the allowed values of r in terms of the quantum number, n .

$$\frac{d}{dr} \left(\frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot M_e \cdot r^2} - \frac{G \cdot M_e \cdot M_s}{r} \right) = 0 \quad \text{has solution(s)} \quad \frac{1}{4} \cdot n^2 \cdot \frac{h^2}{G \cdot [M_e^2 \cdot (M_s \cdot \pi^2)]}$$

Substitution of this value of r in the total energy expression yields the energy of the earth-sun system as a function of the quantum number, n , Planck's constant, the gravitational constant, and the masses of the earth and the sun.

$$E = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot M_e \cdot r^2} - \frac{G \cdot M_e \cdot M_s}{r} \quad \text{by substitution, yields} \quad E = \frac{-2}{n^2 \cdot h^2} \cdot \pi^2 \cdot M_e^3 \cdot G^2 \cdot M_s^2$$

Given the radius of the earth's orbit listed above, calculate the earth's quantum number.

$$r = \frac{1}{4} \cdot n^2 \cdot \frac{h^2}{G \cdot [M_e^2 \cdot (M_s \cdot \pi^2)]} \quad \text{has solution(s)} \quad \left(\begin{array}{l} \frac{-2}{h} \cdot \sqrt{G \cdot M_e \cdot \sqrt{M_s \cdot \pi \cdot \sqrt{r}}} \\ \frac{2}{h} \cdot \sqrt{G \cdot M_e \cdot \sqrt{M_s \cdot \pi \cdot \sqrt{r}}} \end{array} \right) = \left(\begin{array}{l} -2.524 \times 10^{74} \\ 2.524 \times 10^{74} \end{array} \right)$$

The positive root for n is used to calculate the energy of the earth-sun system.

$$n := 2.524 \cdot 10^{74} \quad E := \frac{-2}{n^2 \cdot h^2} \cdot \pi^2 \cdot M_e^3 \cdot G^2 \cdot M_s^2 \quad E = -2.65 \times 10^{33} \text{ J}$$

According to the virial theorem the classical expression for the energy of the earth-sun system with earth orbit radius r is half the potential energy. Note that this gives a value which is in agreement with the Bohr model for the earth-sun system. Is this a legitimate example of the correspondence principle?

$$E := -\frac{G \cdot M_e \cdot M_s}{2 \cdot r} \quad E = -2.65 \times 10^{33} \text{ J}$$

*Johnson and Pedersen, *Problems and Solutions in Quantum Chemistry and Physics*, pages 26-27.