A deBroglie-Bohr Model for Positronium

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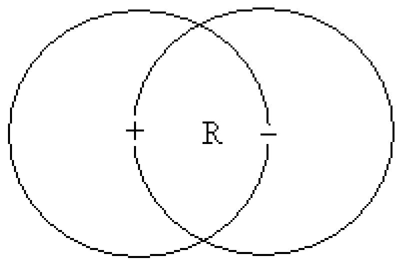
Positronium is a metastable bound state consisting of an electron and its positron antiparticle. In other words it might be thought of as a hydrogen atom in which the proton is replaced by a positron. Naturally it decays quickly after formation due to electron-positron annihilation.

However, it exists long enough for its ground state energy, -0.25 E_h, to be determined. The purpose of this tutorial is to calculate this value using the Bohr model for positronium shown below.



The electron **occupies** a circular orbit of radius R which has a positron at its center. Likewise the positron **occupies** a circular orbit of radius R which has an electron at its center. Occupies has been emphasized to stress that there is no motion, no orbiting. Both particles are behaving as waves (this is the meaning of wave-particle duality) occupying the orbit. As waves they are subject to interference, and to avoid destructive interference the wavelength for the ground state is one orbit circumference.

$$\lambda = 2\cdot\pi\cdot R$$

Introducing the de Broglie relationship between wavelength and momentum, $\lambda = \frac{h}{p}$, yields the following expression for momentum in atomic units (h = 2π).

$$p = \frac{h}{2\cdot\pi\cdot R} = \frac{1}{R}$$

In atomic units m_e = m_p = 1. Therefore, the kinetic energy of each particle is,

$$T = \frac{p^2}{2\cdot m} = \frac{1}{2\cdot R^2}$$

The total energy of positronium is the sum of electron and positron kinetic energies and their coulombic potential energy.

$$E = T_e + T_p + V_{ep} = \frac{1}{2\cdot R^2} + \frac{1}{2\cdot R^2} - \frac{1}{R} = \frac{1}{R^2} - \frac{1}{R}$$

Energy minimization with respect to the electron-positron distance R yields the following result.

$$\frac{d}{dR}\left(\frac{1}{R^2} - \frac{1}{R}\right) = 0 \text{ solve}, R \;\rightarrow 2$$

The optimum R value yields a ground state energy of -0.25 Eh, in agreement with experiment.

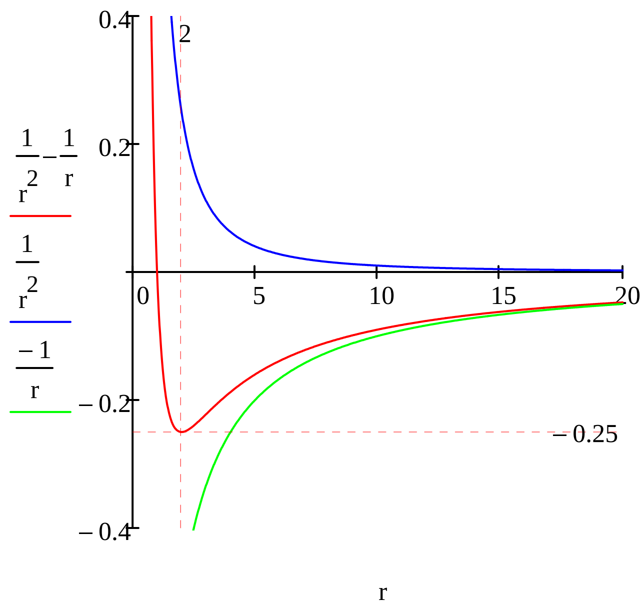
$$E = \frac{1}{R^2} - \frac{1}{R} \text{ substitute}, R = 2 \;\rightarrow E = -\frac{1}{4}$$

Including the symbols for mass in the kinetic energy contributions facilitates the introduction of the concept of effective mass of the composite system.

$$T = \frac{1}{2 \cdot m_e \cdot R^2} + \frac{1}{2 \cdot m_p \cdot R^2} = \frac{1}{2 \cdot R^2} \cdot \left(\frac{1}{m_e} + \frac{1}{m_p} \right) = \frac{1}{2 \cdot R^2} \cdot \left(\frac{m_e + m_p}{m_e \cdot m_p} \right) = \frac{1}{2 \cdot \mu_{ep} \cdot R^2}$$

$$\mu_{ep} = \frac{m_e \cdot m_p}{m_e + m_p} = \frac{1 \cdot 1}{1 + 1} = \frac{1}{2}$$

The positronium energy minimum can also be located graphically.



Plotting kinetic and potential energy along with total energy reveals that a ground state is achieved because beginning at R = 2, the kinetic energy is approaching positive infinity more quickly than the potential energy is approaching negative infinity.