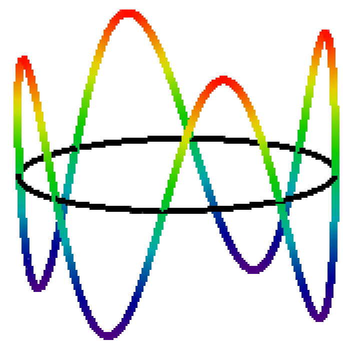
The deBroglie-Bohr Model for a Hydrogen Atom Held Together by a Gravitational Interaction

$\lambda = \frac{h}{m \cdot v}$ de Broglie's hypothesis that matter has wave-like properties.



$n \cdot \lambda = 2 \cdot \pi \cdot r$ The consequence of de Broglie's hypothesis; an integral number of wavelengths must fit within the circumference of the orbit. This introduces the quantum number, n, which can have values 1,2,3,...

$m \cdot v = \frac{n \cdot h}{2 \cdot \pi \cdot r}$ Substitution of the first equation into the second equation reveals that linear momentum is quantized.

$T = \frac{1}{2} \cdot m \cdot v^2 = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2}$ If momentum is quantized, so is kinetic energy.

$E = T + V = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2} - \frac{G \cdot m_p \cdot m_e}{r}$ Which means that total energy is quantized, where $-\frac{G \cdot m_p \cdot m_e}{r}$ is the gravitational potential energy interaction between a proton and an electron.

$$\frac{d}{dr}\left(\frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2} - \frac{G \cdot m_p \cdot m_e}{r}\right) = 0 \text{ solve}, r \rightarrow \frac{h^2 \cdot n^2}{4 \cdot \pi^2 \cdot G \cdot m_e^2 \cdot m_p}$$

Minimization of the energy with respect to orbit radius yields the optimum values of r. This expression is subtituted back in the energy expression below to find the allowed energies.

$$E = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2} - \frac{G \cdot m_p \cdot m_e}{r} \text{ substitute}, r = \frac{h^2 \cdot n^2}{4 \cdot \pi^2 \cdot G \cdot m_e^2 \cdot m_p} \rightarrow E = -\frac{2 \cdot \pi^2 \cdot G^2 \cdot m_e^3 \cdot m_p^2}{h^2 \cdot n^2}$$

Fundamental constants: $m_p := 1.67262 \cdot 10^{-27} \cdot kg$ $m_e := 9.10939 \cdot 10^{-31} \cdot kg$

$h := 6.62608 \cdot 10^{-34} \cdot joule \cdot sec$ $G := 6.67259 \cdot 10^{-11} \cdot \frac{m^3}{kg \cdot s^2}$

Energy: $E(n) := -\frac{2 \cdot \pi^2 \cdot G^2 \cdot m_e^3 \cdot m_p^2}{h^2 \cdot n^2}$ Orbit radius: $r(n) := \frac{h^2 \cdot n^2}{4 \cdot \pi^2 \cdot G \cdot m_e^2 \cdot m_p}$

Calculate first four energy levels and orbit radii.

$n := 1 .. 4$

$$\frac{E(n)}{J} = \begin{pmatrix} -4.233 \times 10^{-97} \\ -1.058 \times 10^{-97} \\ -4.704 \times 10^{-98} \\ -2.646 \times 10^{-98} \end{pmatrix} \qquad \frac{r(n)}{m} = \begin{pmatrix} 1.201 \times 10^{29} \\ 4.803 \times 10^{29} \\ 1.081 \times 10^{30} \\ 1.921 \times 10^{30} \end{pmatrix}$$

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