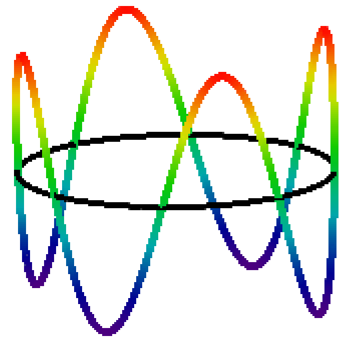
The de Broglie-Bohr Model for the Hydrogen Atom

$$\lambda = \frac{h}{m \cdot v}$$

de Broglie's hypothesis that matter has wave-like properties.

$$n \cdot \lambda = 2 \cdot \pi \cdot r$$

The consequence of de Broglie's hypothesis; an integral number of wavelengths must fit within the circumference of the orbit. This introduces the quantum number, n, which can have values 1,2,3,...



$$m \cdot v = \frac{n \cdot h}{2 \cdot \pi \cdot r}$$

Substitution of the first equation into the second equation reveals that linear momentum is quantized.

$$T = \frac{1}{2} \cdot m \cdot v^2 = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2}$$

If momentum is quantized, so is kinetic energy.

$$E = T + V = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2} - \frac{q^2}{4 \cdot \pi \cdot \varepsilon o_o \cdot r}$$

Which means that total energy is quantized.

Below the ground state energy and orbit radius of the electron in the hydrogen atom is found by plotting the energy as a function of the orbital radius. The ground state is the minimum in the curve.

Fundamental constants: electron charge, electron mass, Planck's constant, vacuum permitivity.

$q := 1.6021777 \cdot 10^{-19} \cdot coul$ $\quad m_e := 9.10939 \cdot 10^{-31} \cdot kg$

$h := 6.62608 \cdot 10^{-34} \cdot joule \cdot sec$ $\quad \varepsilon_o := 8.85419 \cdot 10^{-12} \cdot \frac{coul^2}{joule \cdot m}$

Conversion factors between meters and picometers and joules and atto joules.

$pm := 10^{-12} \cdot m$ $\quad ajoule := 10^{-18} \cdot joule$ $\quad eV := 1.602177 \cdot 10^{-19} \cdot joule$

Setting the first derivative of the energy with respect to r equal to zero, yields the optimum value of r.

$$\frac{d}{dr}\left(\frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2} - \frac{q^2}{4 \cdot \pi \cdot \varepsilon_o \cdot r}\right) = 0$$

has solution(s)

$$n^2 \cdot h^2 \cdot \frac{\varepsilon_o}{q^2 \cdot \pi \cdot m_e}$$

Substitution of this value of r back into the energy expression yields the energy gives the energy of the hydrogen atom in terms of the quantum number, n, and the fundamental constants.

$$E = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2} - \frac{q^2}{4 \cdot \pi \cdot \varepsilon_o \cdot r}$$

by substitution, yields

$$E = \frac{-1}{8 \cdot n^2 \cdot h^2} \cdot \frac{m_e}{\varepsilon_o^2} \cdot q^4$$

Calculate the allowed energy levels for the hydrogen atom: $\quad n := 1 .. 5$

$$E_n := \frac{-1}{8 \cdot n^2 \cdot h^2} \cdot \frac{m_e}{\varepsilon_o^2} \cdot q^4$$

$$\frac{E_n}{ajoule} = \begin{pmatrix} -2.18 \\ -0.545 \\ -0.242 \\ -0.136 \\ -0.087 \end{pmatrix} \qquad \frac{E_n}{eV} = \begin{pmatrix} -13.606 \\ -3.401 \\ -1.512 \\ -0.85 \\ -0.544 \end{pmatrix}$$

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