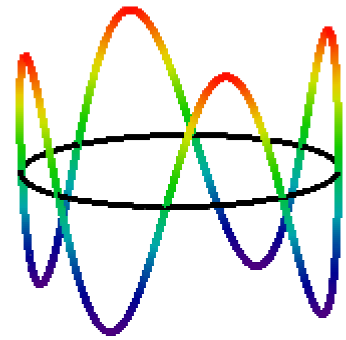
The de Broglie-Bohr Model for the Hydrogen Atom

$\lambda = \frac{h}{m \cdot v}$ — de Broglie's hypothesis that matter has wave-like properties.



$n \cdot \lambda = 2 \cdot \pi \cdot r$ — The consequence of de Broglie's hypothesis; an integral number of wavelengths must fit within the circumference of the orbit. This introduces the quantum number which can have values 1,2,3,...

$m \cdot v = \frac{n \cdot h}{2 \cdot \pi \cdot r}$ — Substitution of the first equation into the second equation reveals that linear momentum is quantized.

$T = \frac{1}{2} \cdot m \cdot v^2 = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2}$ — If momentum is quantized, so is kinetic energy.

$E = T + V = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2} - \frac{e^2}{4 \cdot \pi \cdot \varepsilon_o \cdot r}$ — Which means that total energy is quantized.

Below the ground state energy and orbit radius of the electron in the hydrogen atom is found by plotting the energy as a function of the orbital radius. The ground state is the minimum in the curve.

Fundamental constants: electron charge, electron mass, Planck's constant, vacuum permitivity.

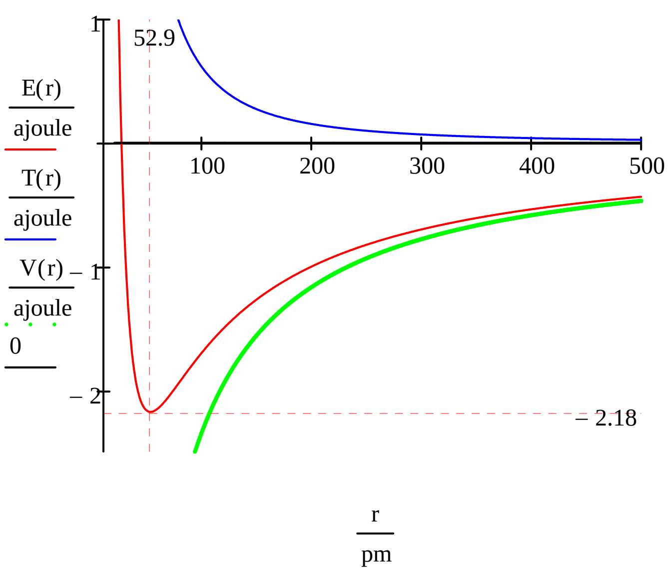
$e := 1.6021777 \cdot 10^{-19} \cdot coul$ $\quad m_e := 9.10939 \cdot 10^{-31} \cdot kg$

$h := 6.62608 \cdot 10^{-34} \cdot joule \cdot sec$ $\quad \varepsilon_0 := 8.85419 \cdot 10^{-12} \cdot \frac{coul^2}{joule \cdot m}$

Quantum number and conversion factor between meters and picometers and joules and atto joules.

$n := 1$ $\quad pm := 10^{-12} \cdot m$ $\quad ajoule := 10^{-18} \cdot joule$

$r := 20 \cdot pm, 20.5 \cdot pm .. 500 \cdot pm$ $\quad T(r) := \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2}$ $\quad V(r) := -\frac{e^2}{4 \cdot \pi \cdot \varepsilon_0 \cdot r}$ $\quad E(r) := T(r) + V(r)$



This figure shows that atomic stability involves a balance between potential and kinetic energy. The electron is drawn toward the nucleus by the attractive potential energy interaction (~ -1/R), but is prevented from spiraling into the nucleus by the extremely large kinetic energy (~1/R^2) associated with small orbits.

Prepared by Frank Rioux.