The Quantum Jump in Momentum Space

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This tutorial is a companion to "The Quantum Jump" which deals with the quantum jump from the perspective of the coordinate-space wave function. This tutorial accomplishes the same thing in momentum space.

The time-dependent momentum wave function for a particle in a one-dimensional box of width $1a_0$ is shown below.

$$\psi(n,p,t) = n \cdot \sqrt{\pi} \cdot \left[\frac{1 - (-1)^n \cdot \exp(-i \cdot p)}{n^2 \cdot \pi^2 - p^2} \right] \cdot \exp(-i \cdot E_i \cdot t)$$

The n = 1 to n = 2 Transition for the Particle in a Box is Allowed

This transition is allowed because it yields a momentum distribution that is asymmetric in time as is shown in the figure below. Consequently it allows for coupling with the perturbing electromagnetic field.

Momentum Increment $P := 100$ Time Increment $T := 100$ Initial State $n_i := 1$ Final State $n_f := 2$

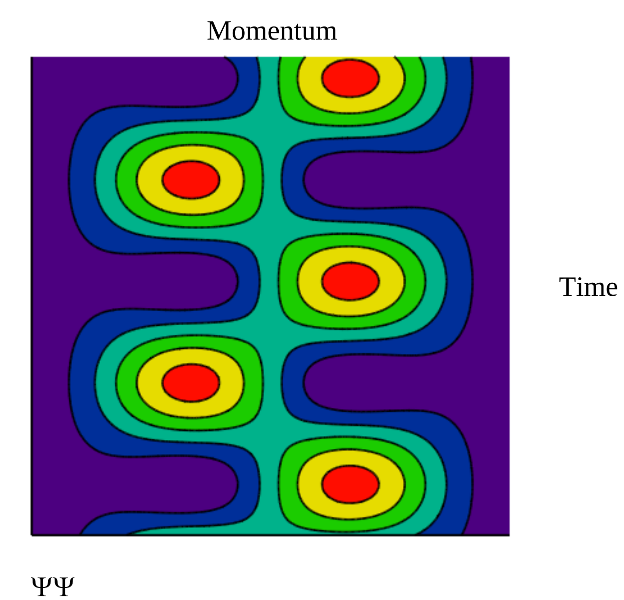
Initial and final energy states for the transition under study: $E_i := \frac{n_i^2 \cdot \pi^2}{2}$ $E_f := \frac{n_f^2 \cdot \pi^2}{2}$

Plot the wavefunction: $j := 0..P$ $p_j := -10 + \frac{20 \cdot j}{P}$ $k := 0..T$ $t_k := \frac{k}{T}$

In the presence of electromagnetic radiation the particle in the box goes into a linear superposition of the stationary states. The linear superpostion for the n = 1 and n = 2 states is given below.

$$\Psi(p,t) := n_i \cdot \sqrt{\pi} \cdot \left[\frac{1 - (-1)^{n_i} \cdot \exp(-i \cdot p)}{n_i^2 \cdot \pi^2 - p^2} \right] \cdot \exp(-i \cdot E_i \cdot t) + n_f \cdot \sqrt{\pi} \cdot \left[\frac{1 - (-1)^{n_f} \cdot \exp(-i \cdot p)}{n_f^2 \cdot \pi^2 - p^2} \right] \cdot \exp(-i \cdot E_f \cdot t)$$

Calculate and plot the momentum distribution: $\Psi^*\Psi$: $\Psi\Psi_{(j,k)} := \left(\left|\Psi(p_j, t_k)\right|\right)^2$



The n = 1 to n = 3 Transition for the Particle in a Box is Not Allowed

This transition is not allowed because it yields a momentum distribution that is symmetric in time as is shown in the figure below. Consequently it does not allow for coupling with the perturbing electromagnetic field.

Momentum Increment $P := 100$ Time Increment $T := 100$ Initial State $n_i := 1$ Final State $n_f := 3$

Initial and final energy states for the transition under study $E_i := \frac{n_i^2 \cdot \pi^2}{2}$ $E_f := \frac{n_f^2 \cdot \pi^2}{2}$

Plot the wavefunction: $j := 0..P$ $p_j := -10 + \frac{20 \cdot j}{P}$ $k := 0..T$ $t_k := \frac{k}{T}$

$$\Psi(p,t) := n_i \cdot \sqrt{\pi} \cdot \left[\frac{1-(-1)^{n_i} \cdot \exp(-i \cdot p)}{n_i^2 \cdot \pi^2 - p^2}\right] \cdot \exp(-i \cdot E_i \cdot t) + n_f \cdot \sqrt{\pi} \cdot \left[\frac{1-(-1)^{n_f} \cdot \exp(-i \cdot p)}{n_f^2 \cdot \pi^2 - p^2}\right] \cdot \exp(-i \cdot E_f \cdot t)$$

In the presence of electromagnetic radiation the particle in the box goes into a linear superposition of the stationary states. The linear superpostion for the n = 1 and n = 3 states is given below.

Calculate and plot the momentum distribution: Ψ*Ψ: $\Psi\Psi_{(j,k)} := \left(\left|\Psi\left(p_j, t_k\right)\right|\right)^2$