Quantum Principles Illuminated with Polarized Light

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When unpolarized light illuminates a polarizing film oriented in the vertical direction 50% of the photons are transmitted. In quantum mechanics this event is called state preparation; the transmitted photons are now in a well-defined state – they are vertically polarized and may be represented by a Dirac ket, $|\updownarrow\rangle$. According to quantum mechanics only two subsequent experiments have certain outcomes.

1. The probability that the vertically polarized photons will pass a second vertical polarizer is 1, $\left(|\langle\updownarrow|\updownarrow\rangle|^2 = 1\right)$.
2. The probability that the vertically polarized photons will pass a second polarizer that is oriented horizontally is 0, $\left(|\langle\leftrightarrow|\updownarrow\rangle|^2 = 0\right)$. In other words, the projection of $|\updownarrow\rangle$ onto $|\leftrightarrow\rangle$ is zero because $|\updownarrow\rangle$ and $|\leftrightarrow\rangle$ are orthogonal.

For all other experiments involving two polarizers only the probability of the outcome can be predicted, and this is $\cos^2(\theta)$, where θ is the relative angle of the polarizing films. See Figure 1 in the appendix for a graphical illustration of the trigonometry involved.

We now proceed to what is usually called the "three polarizer paradox." With two polarizers opposed in the vertical and horizontal orientations, a third polarizer is inserted between them at a 45^o angle. Now some light is transmitted by the final horizontal polarizer. The quantum mechanical interpretation of this experiment is based on the superposition principle and is outlined below.

A vertically polarized photon can be represented as a linear superposition of any other set of orthogonal basis states, for example $\pm 45^o$ relative to the vertical.

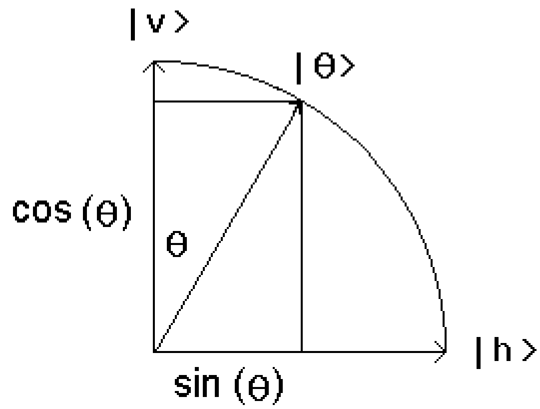
$$|\updownarrow\rangle = \frac{1}{\sqrt{2}}\left[|\nwarrow\!\!\searrow\rangle + |\swarrow\!\!\nearrow\rangle\right]$$

Thus, a vertically polarized photon has a probability of ½ $\left(|\langle\swarrow\!\!\nearrow|\updownarrow\rangle|^2 = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}\right)$ of passing a polarizer oriented at a 45^o angle. A photon that has passed a 45^o polarizer is in the state $|\swarrow\!\!\nearrow\rangle$. This state can be written as a linear superposition of a vertically and horizontally polarized photon.

$$|\swarrow\!\!\nearrow\rangle = \frac{1}{\sqrt{2}}\left[|\updownarrow\rangle + |\leftrightarrow\rangle\right]$$

Photons that have passed the 45° polarizer have a probability of ½ $\left(\left|\langle\leftrightarrow|\swarrow\!\!\nearrow\rangle\right|^2 = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}\right)$ of passing the final horizontal polarizer. To summarize, in the absence of the diagonally oriented polarizer none of the original unpolarized photons pass the final horizontal polarizer, but in its presence 12.5% of the photons are transmitted (½½½). See the second figure in the appendix for a graphical representation of the three-polarizer demonstration.

Appendix



The probability amplitude that a θ-polarized photon will pass a vertical polarizer is $\langle v|\theta\rangle = \cos(\theta)$. The probability for this event, therefore, is $\left|\langle v|\theta\rangle\right|^2 = \cos^2(\theta)$.

Figure 1

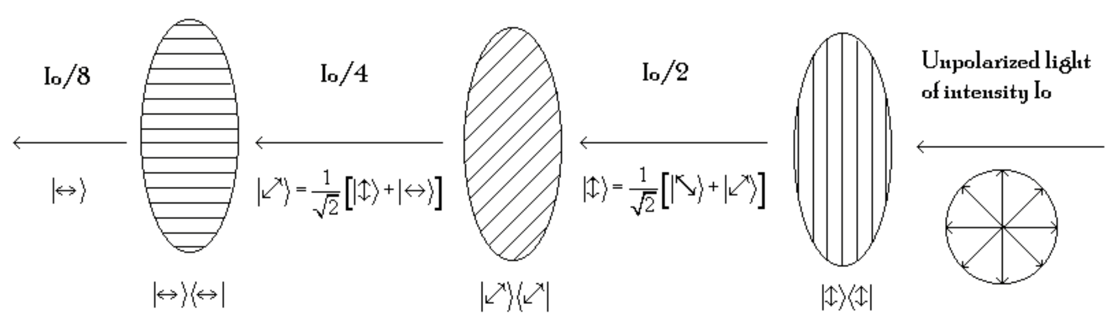


Figure 2