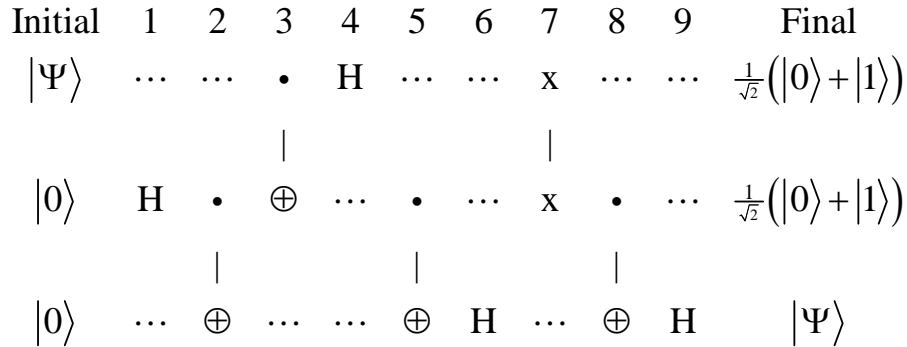


Yet Another Quantum Teleportation Circuit

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This quantum teleportation circuit transfers Ψ from the top wire on the left to the lower wire on the right without the requirement of measurements on the top two wires, which are occupied by identical superposition states.



The required matrix operators:

$$I := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad H := \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \text{CNOT} := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{Swap} := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The initial state:

$$\Psi = \begin{pmatrix} \sqrt{\frac{1}{3}} \\ \sqrt{\frac{2}{3}} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \Psi := \left(\sqrt{\frac{1}{3}} \ 0 \ 0 \ 0 \ \sqrt{\frac{2}{3}} \ 0 \ 0 \ 0 \right)^T$$

Because of its length, the quantum circuit is written in steps.

- | | | |
|--|-----------------------------|--|
| Step1 := kronecker(I, kronecker(H, I)) | Step2 := kronecker(I, CNOT) | Step3 := kronecker(CNOT, I) |
| Step4 := kronecker(H, kronecker(I, I)) | Step5 := kronecker(I, CNOT) | Step6 := kronecker(I, kronecker(I, H)) |
| Step7 := kronecker(Swap, I) | Step8 := kronecker(I, CNOT) | Step9 := kronecker(I, kronecker(I, H)) |

$$\text{QC} := \text{Step9} \cdot \text{Step8} \cdot \text{Step7} \cdot \text{Step6} \cdot \text{Step5} \cdot \text{Step4} \cdot \text{Step3} \cdot \text{Step2} \cdot \text{Step1}$$

The quantum circuit operates on the initial state, transferring Ψ from the top wire to the bottom wire.

$$\text{QC} \cdot \Psi = \begin{pmatrix} 0.289 \\ 0.408 \\ 0.289 \\ 0.408 \\ 0.289 \\ 0.408 \\ 0.289 \\ 0.408 \end{pmatrix} = \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} \sqrt{\frac{1}{3}} \\ \sqrt{\frac{2}{3}} \end{pmatrix} = \frac{1}{2} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} \sqrt{\frac{1}{3}} \\ \sqrt{\frac{2}{3}} \end{pmatrix} = \begin{pmatrix} 0.289 \\ 0.408 \\ 0.289 \\ 0.408 \\ 0.289 \\ 0.408 \\ 0.289 \\ 0.408 \end{pmatrix} \quad \frac{1}{2} \cdot \sqrt{\frac{1}{3}} = 0.289 \quad \frac{1}{2} \cdot \sqrt{\frac{2}{3}} = 0.408$$