Yet Another Quantum Teleportation Circuit

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This quantum teleportation circuit transfers Ψ from the top wire on the left to the lower wire on the right without the requirement of measurements on the top two wires, which are occupied by identical superposition states.

Initial	1	2	3	4	5	6	7	8	9	Final
$\vert\Psi\rangle$	⋯	⋯	•	H	⋯	⋯	x	⋯	⋯	$\frac{1}{\sqrt{2}}(\vert 0\rangle + \vert 1\rangle)$
			|				|			
$\vert 0\rangle$	H	•	⊕	⋯	•	⋯	x	•	⋯	$\frac{1}{\sqrt{2}}(\vert 0\rangle + \vert 1\rangle)$
		|			|			|		
$\vert 0\rangle$	⋯	⊕	⋯	⋯	⊕	H	⋯	⊕	H	$\vert\Psi\rangle$

The required matrix operators:

$$I := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad H := \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad CNOT := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad Swap := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The initial state:

$$\Psi = \begin{pmatrix} \sqrt{\frac{1}{3}} \\ \sqrt{\frac{2}{3}} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \Psi := \begin{pmatrix} \sqrt{\frac{1}{3}} & 0 & 0 & 0 & \sqrt{\frac{2}{3}} & 0 & 0 & 0 \end{pmatrix}^T$$

Because of its length, the quantum circuit is written in steps.

$Step1 := kronecker(I, kronecker(H, I))$ $\quad Step2 := kronecker(I, CNOT)$ $\quad Step3 := kronecker(CNOT, I)$

$Step4 := kronecker(H, kronecker(I, I))$ $\quad Step5 := kronecker(I, CNOT)$ $\quad Step6 := kronecker(I, kronecker(I, H))$

$Step7 := kronecker(Swap, I)$ $\quad Step8 := kronecker(I, CNOT)$ $\quad Step9 := kronecker(I, kronecker(I, H))$

$$QC := Step9 \cdot Step8 \cdot Step7 \cdot Step6 \cdot Step5 \cdot Step4 \cdot Step3 \cdot Step2 \cdot Step1$$

The quantum circuit operates on the initial state, transfering Ψ from the top wire to the bottom wire.

$$QC \cdot \Psi = \begin{pmatrix} 0.289 \\ 0.408 \\ 0.289 \\ 0.408 \\ 0.289 \\ 0.408 \\ 0.289 \\ 0.408 \end{pmatrix} \qquad \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} \sqrt{\frac{1}{3}} \\ \sqrt{\frac{2}{3}} \end{pmatrix} = \frac{1}{2} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} \sqrt{\frac{1}{3}} \\ \sqrt{\frac{2}{3}} \end{pmatrix} = \begin{pmatrix} 0.289 \\ 0.408 \\ 0.289 \\ 0.408 \\ 0.289 \\ 0.408 \\ 0.289 \\ 0.408 \end{pmatrix} \qquad \frac{1}{2} \cdot \sqrt{\frac{1}{3}} = 0.289 \qquad \frac{1}{2} \cdot \sqrt{\frac{2}{3}} = 0.408$$