A Quantum Teleportation Experiment for Undergraduates

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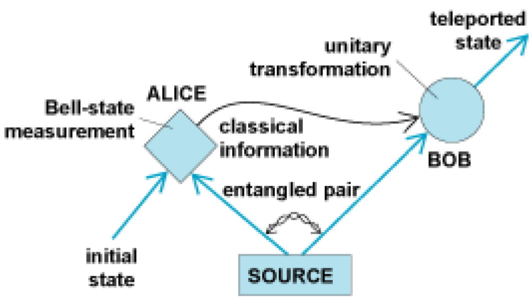
This Mathcad document examines the math involved in a teleportation experiment for undergraduates using IBM's 5-qubit quantum processor (IBM Quantum Experience) posted by S. Fedortchenko at arXiv:1607.02398v1. Except for state preparation it is identical to the following teleportation circuit, which can be found in my "Teleportation: Another Look."



	Initial						Final
	$\lvert\Psi\rangle$	⋯	•	⋯	$\boxed{H}$	▷	Measure $\lvert a\rangle$ 0 or 1
Alice			|				Bell state measurement
	•	⋯	⊕	⋯	⋯	▷	Measure $\lvert b\rangle$ 0 or 1
	β_{00}						
Bob	•	⋯	⋯	⋯	⋯	▷	$X^b Z^a \rightarrow \lvert\Psi\rangle$

Fedortchenko's teleportation circuit is shown below.

Initial		1	2	3	4	5	6		Final
$\lvert 0\rangle$	▷	$\boxed{H}$	$\boxed{T}$	$\boxed{H}$	$\boxed{S}$	•	$\boxed{H}$	▷	Measure $\lvert a\rangle$ 0 or 1
						|			Bell state measurement
$\lvert 0\rangle$	▷	⋯	⋯	⋯	⊕	⊕	⋯	▷	Measure $\lvert b\rangle$ 0 or 1
					|				
$\lvert 0\rangle$	▷	⋯	⋯	$\boxed{H}$	•	⋯	⋯	▷	$X^b Z^a \rightarrow \lvert\Psi\rangle$

Single qubit operators:

$$I := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad H := \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad S := \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \quad T := \begin{pmatrix} 1 & 0 \\ 0 & e^{i \cdot \frac{\pi}{4}} \end{pmatrix} \quad X := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Z := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Two qubit operators:

$$CNOT := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad ICNOT := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Demonstrate the generation of the teleportee state held by Alice:

$$S \cdot H \cdot T \cdot H \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.854 + 0.354i \\ 0.354 + 0.146i \end{pmatrix} \qquad e^{i \cdot \frac{\pi}{8}} \cdot \left[\cos\left(\frac{\pi}{8}\right) \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \sin\left(\frac{\pi}{8}\right) \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] = \begin{pmatrix} 0.854 + 0.354i \\ 0.354 + 0.146i \end{pmatrix}$$

Demonstrate the creation of the entangled Bell state shared by Alice and Bob:

$$\text{ICNOT}\cdot(\text{kronecker}(I,H))\begin{pmatrix}1\\0\\0\\0\end{pmatrix}=\begin{pmatrix}0.707\\0\\0\\0.707\end{pmatrix} \qquad \frac{1}{\sqrt{2}}\cdot\left[\begin{pmatrix}1\\0\end{pmatrix}\cdot\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}\cdot\begin{pmatrix}0\\1\end{pmatrix}\right]$$

Creation of the teleportee state and the entangled Bell state occurs in the first four steps.

$$\text{StatePrep} := \text{kronecker}(S,\text{ICNOT})\cdot\text{kronecker}(H,\text{kronecker}(I,H))\cdot\text{kronecker}(T,\text{kronecker}(I,I))\cdot\text{kronecker}(H,\text{kronecker}(I,I))$$

Teleportation occurs in steps 5 and 6.

$$\text{TC} := \text{kronecker}(H,\text{kronecker}(I,I))\cdot\text{kronecker}(\text{CNOT},I)$$

$$\text{TC}\cdot\text{StatePrep}\cdot\begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix}=\begin{pmatrix}0.427+0.177i\\0.177+0.073i\\0.177+0.073i\\0.427+0.177i\\0.427+0.177i\\-0.177-0.073i\\-0.177-0.073i\\0.427+0.177i\end{pmatrix}$$

Measurement occurs in the final step on the top two wires with possible outcomes |00>, |01>, |10> and |11>. In other words, Alice makes a Bell state measurement (see first figure above) on the two qubits in her possession and informs Bob of the result through a classical channel. He then performs an operation on his qubit to recover the teleported state.

If Alice observes |00> Bob does nothing (the identity operation) because he has the teleportee on his register. If Alice observes |01> Bob applies the X operator, if she finds |10> he uses the Z operator, and finally if Alice observes |11> Bob applies the X operator followed by the Z operator. Further mathematical detail is provided by showing explicitly the four equally probable measurement outcomes that Alice observes, and Bob's subsequent action on his register.

Computational Details

Measurement operator for |0>: $\begin{pmatrix}1\\0\end{pmatrix}\cdot\begin{pmatrix}1 & 0\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$ Measurement operator for |1>: $\begin{pmatrix}0\\1\end{pmatrix}\cdot\begin{pmatrix}0 & 1\end{pmatrix} = \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$

Alice measures |00>: $\begin{pmatrix}1\\0\end{pmatrix}\cdot\begin{pmatrix}1\\0\end{pmatrix}\cdot\begin{pmatrix}0.854 + 0.354i\\0.354 + 0.146i\end{pmatrix}$ Bob's action: $I\cdot\begin{pmatrix}0.854 + 0.354i\\0.354 + 0.146i\end{pmatrix} = \begin{pmatrix}0.854 + 0.354i\\0.354 + 0.146i\end{pmatrix}$

$$2\cdot\text{kronecker}\left[\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}, \text{kronecker}\left[\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}, I\right]\right]\cdot TC\cdot StatePrep\cdot\begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} = \begin{pmatrix}0.854 + 0.354i\\0.354 + 0.146i\\0\\0\\0\\0\\0\\0\end{pmatrix}$$

Alice measures |01>: $\begin{pmatrix}1\\0\end{pmatrix}\cdot\begin{pmatrix}0\\1\end{pmatrix}\cdot\begin{pmatrix}0.354 + 0.146i\\0.854 + 0.354i\end{pmatrix}$ Bob's action: $X\cdot\begin{pmatrix}0.354 + 0.146i\\0.854 + 0.354i\end{pmatrix} = \begin{pmatrix}0.854 + 0.354i\\0.354 + 0.146i\end{pmatrix}$

$$2\cdot\text{kronecker}\left[\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}, \text{kronecker}\left[\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}, I\right]\right]\cdot TC\cdot StatePrep\cdot\begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0.354 + 0.146i\\0.854 + 0.354i\\0\\0\\0\\0\end{pmatrix}$$

Alice measures |10>: $\begin{pmatrix}0\\1\end{pmatrix}\cdot\begin{pmatrix}1\\0\end{pmatrix}\cdot\begin{pmatrix}0.854 + 0.354i\\-0.354 - 0.146i\end{pmatrix}$ Bob's action: $Z\cdot\begin{pmatrix}0.854 + 0.354i\\-0.354 - 0.146i\end{pmatrix} = \begin{pmatrix}0.854 + 0.354i\\0.354 + 0.146i\end{pmatrix}$

$$2\cdot\text{kronecker}\left[\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}, \text{kronecker}\left[\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}, I\right]\right]\cdot TC\cdot StatePrep\cdot\begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\\0.854 + 0.354i\\-0.354 - 0.146i\\0\\0\end{pmatrix}$$

Alice measures |11>: $\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -0.354 - 0.146i \\ 0.854 + 0.354i \end{pmatrix}$ Bob's action: $Z \cdot X \cdot \begin{pmatrix} -0.354 - 0.146i \\ 0.854 + 0.354i \end{pmatrix} = \begin{pmatrix} 0.854 + 0.354i \\ 0.354 + 0.146i \end{pmatrix}$

$$2 \cdot \text{kronecker}\left[\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \text{kronecker}\left[\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, I\right]\right] \cdot TC \cdot StatePrep \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0.354 - 0.146i \\ 0.854 + 0.354i \end{pmatrix}$$