The Ramsey Atomic Interferometer

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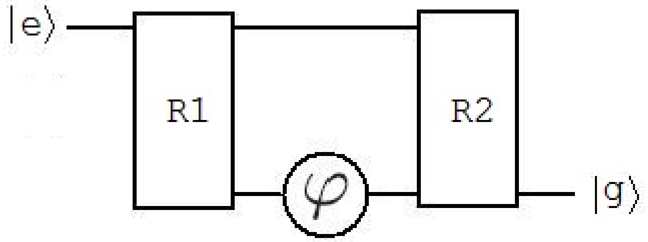
The Ramsey interferometer, which closely resembles the Mach-Zehnder interferometer, is constructed using two $\pi/2$ Rabi pulses (R1 and R2) separated by a phase shifter in the lower arm, as shown below.



where $|e\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad |g\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

The matrix representations of the phase shifter and the Rabi elements are as follows:

$$\text{PhaseShift}(\phi) := \begin{pmatrix} 1 & 0 \\ 0 & e^{i\cdot\phi} \end{pmatrix} \qquad \text{Rabi}(\theta) := \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & -\sin\left(\frac{\theta}{2}\right) \\ \sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{pmatrix} \qquad \text{Rabi}\left(\frac{\pi}{2}\right) = \begin{pmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{pmatrix}$$

The input to the interferometer is the upper state |e> of a two-state atom. The first pulse behaves like a Hadamard gate creating a coherent superposition of |e> and the lower state of the atom, |g>.

$$\text{Rabi}\left(\frac{\pi}{2}\right)\cdot\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{2}\cdot 2^{\frac{1}{2}} \\ \frac{1}{2}\cdot 2^{\frac{1}{2}} \end{pmatrix} \qquad \frac{1}{\sqrt{2}}\cdot\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

The phase shifter alters the superposition by adding a phase to |g>.

$$\text{PhaseShift}(\phi)\cdot\text{Rabi}\left(\frac{\pi}{2}\right)\cdot\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{2}\cdot 2^{\frac{1}{2}} \\ \frac{1}{2}\cdot e^{i\cdot\phi}\cdot 2^{\frac{1}{2}} \end{pmatrix} \qquad \frac{1}{\sqrt{2}}\cdot\begin{pmatrix} 1 \\ e^{i\cdot\phi} \end{pmatrix}$$

In the absence of a phase shift ($\phi = 0$) the two $\pi/2$ pulses behave like a not gate yielding |g> at the output channel.

$$\text{Rabi}\left(\frac{\pi}{2}\right)\cdot\text{PhaseShift}(0)\cdot\text{Rabi}\left(\frac{\pi}{2}\right)\cdot\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

However if $\phi = \pi$, the interferometer is equivalent to the identity operator and the output is |e>.

$$\text{Rabi}\left(\frac{\pi}{2}\right)\cdot\text{PhaseShift}(\pi)\cdot\text{Rabi}\left(\frac{\pi}{2}\right)\cdot\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

In general, the result is a superposition of |e> and |g>.

$$\mathrm{Rabi}\left(\frac{\pi}{2}\right)\cdot \mathrm{PhaseShift}(\phi)\cdot \mathrm{Rabi}\left(\frac{\pi}{2}\right)\cdot\begin{pmatrix}1\\0\end{pmatrix} \rightarrow \begin{pmatrix}\frac{1}{2}-\frac{1}{2}\cdot e^{i\cdot\phi}\\ \frac{1}{2}+\frac{1}{2}\cdot e^{i\cdot\phi}\end{pmatrix} \qquad \begin{pmatrix}e\\g\end{pmatrix} = \frac{1}{2}\cdot\begin{pmatrix}1-e^{i\cdot\phi}\\1+e^{i\cdot\phi}\end{pmatrix}$$

The probabilities of detecting |e> and |g> at the output channel depend on the phase ϕ, exhibiting interference effects as in the Mach-Zehnder interferometer with a phase shifter in the lower arm. This is shown below both algebraically and graphically.

$$P_e(\phi) := \left[\left|\begin{pmatrix}1\\0\end{pmatrix}^T\cdot \mathrm{Rabi}\left(\frac{\pi}{2}\right)\cdot \mathrm{PhaseShift}(\phi)\cdot \mathrm{Rabi}\left(\frac{\pi}{2}\right)\cdot\begin{pmatrix}1\\0\end{pmatrix}\right|\right]^2 \text{ simplify } \rightarrow \frac{1}{2}-\frac{1}{2}\cdot\cos(\phi)$$

$$P_g(\phi) := \left[\left|\begin{pmatrix}0\\1\end{pmatrix}^T\cdot \mathrm{Rabi}\left(\frac{\pi}{2}\right)\cdot \mathrm{PhaseShift}(\phi)\cdot \mathrm{Rabi}\left(\frac{\pi}{2}\right)\cdot\begin{pmatrix}1\\0\end{pmatrix}\right|\right]^2 \text{ simplify } \rightarrow \frac{1}{2}+\frac{1}{2}\cdot\cos(\phi)$$

$$\phi := 0, .01\cdot\pi .. 4\cdot\pi$$