Quantum Computation: A Short Course

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> The reason I was keen to include at least some mathematical descriptions was simply that in my own study of quantum computation the only time I *really* felt that I understood what was happening in a quantum program was when I examined some typical quantum circuits and followed through the equations. Julian Brown, *The Quest for the Quantum Computer,* page 6.

My reason for beginning with Julian Brown's statement is that I accept it wholeheartedly. I learn the same way. So in what follows I will present mathematical analyses of some relatively simple and representative quantum circuits that are designed to carry out important contemporary processes such as parallel computation, teleportation, data-base searches, prime factorization, quantum encryption and quantum simulation. I will conclude with a foray into the related area of Bell's theorem and the battle between local realism and quantum mechanics.

Quantum computers use superpositions, entanglement and interference to carry out calculations that are impossible with a classical computer. The following link contains insightful descriptions of the non-classical character of superpositions and entangled superpositions from a variety of sources.

http://www.users.csbsju.edu/~frioux/q-intro/EntangledSuperposition.pdf

To illuminate the difference between classical and quantum computation we begin with a review of the fundamental principles of quantum theory using the computational methods of matrix mechanics.

http://www.users.csbsju.edu/~frioux/matmech/RudimentaryMatrixMechanics.pdf

The following is an archive of photon and spin vector states and their matrix operators.

http://www.users.csbsju.edu/~frioux/q-intro/MatrixEigenstatesOperators.pdf

The characteristic feature of a quantum computer is its ability to calculate in parallel. How this is accomplished is illustrated in the following one-page tutorial.

http://www.users.csbsju.edu/~frioux/q-intro/QuantumComputerIntro.pdf

The following tutorial adds a matrix analysis to the previous example of parallel calculation.

http://www.users.csbsju.edu/~frioux/q-intro/SimpleParallelCalculation.pdf

A slightly more complicated example of quantum parallel computation is provided in the next tutorial.

http://www.users.csbsju.edu/~frioux/q-intro/QuantumParallelComputation.pdf

Solving systems of equations is a relatively routine task for a quantum circuit.

http://www.users.csbsju.edu/~frioux/q-intro/QC-SolveEqBrief.pdf

One of the most intriguing applications of entanglement is quantum teleportation, which uses entanglement and a classical communication channel to transfer a quantum state from one location to another. However, to truly understand teleportation it is necessary to distinguish it from cloning. So first we look at the quantum no-cloning principle followed by a one-page snapshot of teleportation.

http://www.users.csbsju.edu/~frioux/q-intro/QuantumClone.pdf

http://www.users.csbsju.edu/~frioux/q-intro/BriefTeleportationSummary.pdf

The following tutorial provides four in-depth perspectives on teleportation.

http://www.users.csbsju.edu/~frioux/q-intro/TeleportationOutline.pdf

As can be seen in these previous tutorials, teleportation involves entanglement transfer. Alice projects her photons onto one of the entangled Bell states and Bob receives a photon state which using information provided via the classical communication channel can be transformed into the teleportee state. Given the importance of entanglement in quantum computing a more elaborate example of entanglement transfer is provided in the following tutorial.

http://www.users.csbsju.edu/~frioux/q-intro/EntanglementSwapBrief.pdf

In our earlier examination of the quantum computer we saw that a quantum circuit can calculate all the values of f(x) simultaneously, but we can only retrieve one value due to the collapse of the superposition of answers on observation. To achieve a quantum advantage in computation requires more subtle programming techniques which exploit the effects of quantum interference. The following tutorials reveal how the quantum advantage can be achieved in several areas of practical importance.

> While quantum mechanics could spell disaster for public-key cryptography, it may also offer salvation. This is because the resources of the quantum world appear to offer the ultimate form of secret code, one that is guaranteed by the laws of physics to be unbreakable. Julian Brown, *The Quest for the Quantum Computer*, page 189.

In other words, “The quantum taketh away and the quantum giveth back!” Asher Peres

We begin with Shor’s algorithm which demonstrates how quantum entanglement and interference effects can facilitate the factorization of large integers into their prime factors. The inability of conventional computers to do this is essential to the integrity of public-key cryptography.

http://www.users.csbsju.edu/~frioux/q-intro/ShorAlgorithmSummary.pdf

How quantum theory gives back is demonstrated by an examination of Ekert’s quantum secret key proposal.

http://www.users.csbsju.edu/~frioux/q-intro/EkertSecretKey.pdf

http://www.users.csbsju.edu/~frioux/q-intro/CodingDecodingVenus.pdf

If you are asked if two pieces of glass have the same thickness, the conventional thing to do is to measure the thickness of each piece – two measurements. As shown in the first two tutorials a double-slit apparatus and a Mach-Zehnder interferometer can answer the question with a single measurement. The third tutorial is a version of the first which shows how path information destroys interference and how the interference can be restored. The fourth tutorial summarizes David Deutsch's solution to an equivalent mathematical problem using a function introduced earlier. The double-slit apparatus, the Mach-Zehnder interferometer and Deutsch's circuit are quantum computers which use superpositions and interference effects to cut the effort of answering the question by a factor of two.

http://www.users.csbsju.edu/~frioux/q-intro/2slit-QuComputer.pdf

http://www.users.csbsju.edu/~frioux/q-intro/QC-MZI.pdf

http://www.users.csbsju.edu/~frioux/two-slit/WhichPathEraser.pdf

http://www.users.csbsju.edu/~frioux/q-intro/DeutschProblemBrief.pdf

The Mach-Zehnder interferometer introduced above can be used to illuminate several contentious issues in quantum mechanics. The sub-microscopic building blocks of the natural world (electrons, protons, neutrons and photons) do not behave like the macroscopic objects we encounter in daily life because they have both wave and particle characteristics. Nick Herbert (Quantum Reality, p. 64) called them quons. "A quon is any entity … that exhibits both wave and particle aspects in the peculiar quantum manner." The 'peculiar quantum manner' is that while we always observe particles, prior to measurement or observation quons behave like waves. This peculiar behavior is illustrated in the following tutorial.

http://www.users.csbsju.edu/~frioux/interfer/FeynmanHistories.pdf

This tutorial and several of the previous ones have used the Mach-Zehnder interferometer to illuminate important quantum concepts. The following tutorial outlines the role of the Michelson interferometer in Fourier Transform spectroscopy.

http://www.users.csbsju.edu/~frioux/nmr/mic-interfer.pdf

Some quantum physicists believe they can isolate wave and particle behavior in a properly designed experiment. They also invoke the concept of delayed choice. I disagree with both in the next tutorial, but first some remarks by John Wheeler. John Wheeler, designer of several delayed-choice experiments (both terrestrial and cosmological), had the following to say about the interpretation of such experiments.

> … in a loose way of speaking, we decide what the photon *shall have done* after it has *already* done it. In actuality it is wrong to talk of the 'route' of the photon. For a proper way of speaking we recall once more that it makes no sense to talk of a phenomenon until it has been brought to a close by an irreversible act of amplification. 'No elementary phenomenon is a phenomenon until it is a registered (observed) phenomenon.'

http://www.users.csbsju.edu/~frioux/q-intro/DelayedChoice.pdf

In summary, I accept the Copenhagen interpretation of quantum mechanics for the reasons so cogently stated by David Lindley on page 164 of *Where Does The Weirdness Go?*

> And since none of the other "interpretations" of quantum mechanics that we have looked at has brought us any real peace of mind, they simply push the weirdness around, from one place to another, but cannot make it go away – let us stick with the Copenhagen interpretation, which has the virtues of simplicity and necessity. It takes quantum mechanics seriously, takes its weird aspects at face value, and provides an economical, austere, perhaps even antiseptic, account of them.

We now turn to some true quantum weirdness. The following tutorial uses a Mach-Zehnder interferometer (MZI) and Feynman's sum over histories approach to demonstrate interaction-free measurement, or how to see in the dark.

http://www.users.csbsju.edu/~frioux/q-intro/InteractionFreeMeasurement.pdf

Perhaps weirder is the quantum Cheshire cat. The following tutorial shows how a MZI can be engineered to create a situation in which a photon is separated from a property, in this case its angular momentum.

http://www.users.csbsju.edu/~frioux/q-intro/CheshireCat.pdf

The last several tutorials were a bit off the theme of quantum computation. We get back on track with a look at data base searching the quantum way, or the best way to find a needle in a hay stack. This is followed by a demonstration of Simon's algorithm, an illustration of quantum dense coding and an example of quantum error correction.

http://www.users.csbsju.edu/~frioux/q-intro/GroverMonroe.pdf

http://www.users.csbsju.edu/~frioux/q-intro/SimonsAlgorithm.pdf

http://www.users.csbsju.edu/~frioux/q-intro/QuantumDenseCoding.pdf

http://www.users.csbsju.edu/~frioux/q-intro/CorrectQuantumError.pdf

And now we move to Bell's theorem and the battle between local realism and quantum mechanics.

> Quantum theory is both stupendously successful as an account of the small-scale structure of the world and it is also the subject of unresolved debate and dispute about its interpretation. J. C. Polkinghorne, *The Quantum World*.

So we end this short course with the conflict between quantum mechanics and the classical view of reality held by Einstein and others known as local realism. Until the work of John Bell this battle was regarded by most scientists as a distracting philosophical debate. According to David Mermin quantum physicists were admonished to ignore the debate and "Shut up and calculate!"

In 1964 Bell demonstrated that experiments were possible for which quantum theory and local realism gave conflicting predictions, thus moving the disagreement from the realm of philosophical debate to the jurisdiction of the laboratory. In *Where Does the Weirdness Go?* David Lindley summarized Bell's achievement.

> The hallmark of Bell's work was his success in locating precisely the point at which classical views of reality ran into trouble with quantum mechanics, and in devising a means by which the two viewpoints could be empirically compared. Bell's sympathies

seem often to have lain with Bohm and the effort to establish a hidden variables version of quantum mechanics. His greatest achievement, though, was to demonstrate incontrovertibly the price that must be paid to make such a theory work.

Bell said that if a theory of reality was local it would not agree with quantum mechanics, and if it agreed with quantum mechanics it would contain non-local interactions, in other words "spooky interactions at a distance" to use Einstein's phrase. Here's the best description of this spookiness that I have read: "A non-local interaction links up one location with another without crossing space, without decay, and without delay. A non-local event is, in short, unmediated, unmitigated and immediate." Nick Herbert, *Quantum Reality*, page 214.

In 1981 Richard Feynman gave a lecture titled "Simulating Nature with Computers." His thesis was that the simulation of nature at the nanoscopic level required a quantum computer. "I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit. And if you want to make a simulation of nature, you'd better make it quantum mechanical …"

The five remaining tutorials deal with the clash between quantum mechanics and local realism, and the simulation of physical phenomena. The first three examine entangled spin systems and the fourth and fifth entangled photon systems. They all show the disagreement between the predictions of quantum theory and those of local hidden-variable models using different experimental perspectives.

http://www.users.csbsju.edu/~frioux/q-intro/BohmEPR-Extended.pdf

http://www.users.csbsju.edu/~frioux/q-intro/EPRBell-Revised.pdf

http://www.users.csbsju.edu/~frioux/q-intro/GHZ-Brief-Simulation.pdf

http://www.users.csbsju.edu/~frioux/q-intro/NewFeynman5.pdf

http://www.users.csbsju.edu/~frioux/q-intro/PositroniumAnnihilationOutline.pdf

I turn to David Lindley again (*Where Does the Weirdness Go?* page 15) for a concluding comment on the issues dealt with in this section.

> We are, through long familiarity, grounded in the assumption of an external, objective, and definite reality, regardless of how much or how little we actually know about it. It is hard to find the language or the concepts to deal with a "reality" that only becomes real when it is measured. There is no easy way to grasp this change of perspective, but persistence and patience allow a certain new familiarity to supplant the old.

Bibliography

- The Quest for the Quantum Computer, Julian Brown
- The Meaning of Quantum Theory, Jim Baggott
- Quantum Reality, Nick Herbert
- Where Does the Weirdness Go?, David Lindley
- The Cosmic Code, Heinz Pagels
- Quantum Mechanics and Experience, David Z Albert
- The Age of Entanglement, Louisa Gilder

- Quantum Mechanics, Alastair Rae
- Quantum Physics: Illusion or Reality, Alastair Rae
- The Quantum Divide, Gerry and Bruno
- Quantum Weirdness, William J. Mullin
- Through Two Doors at Once, Anil Ananthaswamy
- Beyond Weird, Philip Ball
- Quantum Physics: What Everyone Needs to Know, Michael G. Raymer
- The Quantum World, J. C. Polkinghorne
- Quantum Theory: A Very Short Introduction, John Polkinghorne
- The Quantum Challenge, Greenstein and Zajonc
- Quantum Enigma, Rosenblum and Kuttner
- Programming the Universe, Seth Lloyd
- The Strange World of Quantum Mechanics, Daniel F. Styer