The Pauli Matrices in Quantum Mechanics

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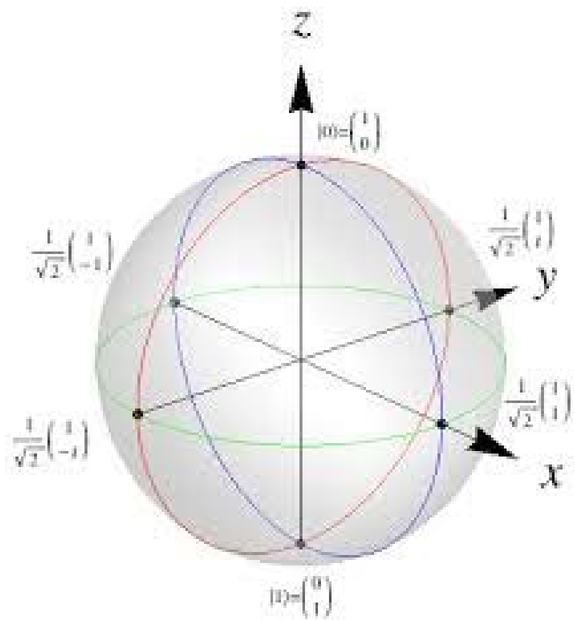
The Pauli matrices or operators are ubiquitous in quantum mechanics. They are most commonly associated with spin ½ systems, but they also play an important role in quantum optics and quantum computing. In what follows, whether we are dealing with electrons, protons or photons, the state vectors we will use or some superposition of them are shown on a Bloch sphere.



We begin by demonstrating how the Pauli matrices can be generated from the state vectors shown above. It is also shown that given the Pauli matrices we can calculate their eigenvalues and eigenvectors and construct the Bloch sphere.

http://www.users.csbsju.edu/~frioux/q-intro/BlochSphereAlt.pdf

To facilitate the understanding of the applications of the Pauli operators an introduction to matrix mechanics is provided by the following link. Here you will find the Pauli matrices temporarily disguised as the operators for some common macroscopic properties such as hardness, color and taste, and learn that quantum candy can't simultaneously be hard, white and sweet.

http://www.users.csbsju.edu/~frioux/matmech/RudimentaryMatrixMechanics.pdf

The following experiment uses three Stern-Gerlach magnets to illustrate the superposition principle. As David Mermin has written, "Superpositions have no classical interpretation. They are *sui generis*, an intrinsically quantum-mechanical construct..."

http://www.users.csbsju.edu/~frioux/q-intro/SternGerlach.pdf

The following well-known demonstration using three polarizing films accomplishes the same objective using light.

http://www.users.csbsju.edu/~frioux/q-intro/polar-append.pdf

More detail about the "three polarizer paradox" is provided in the following tutorial.

http://www.users.csbsju.edu/~frioux/polarize/MatMechPolar.pdf

Staying with light, the Pauli spin operator in the y-direction is essential in understanding optical activity because it is the photon angular momentum operator.

http://www.users.csbsju.edu/~frioux/q-intro/OpticalActivity.pdf

The details of atomic structure are covered next with a look the hyperfine interaction and spin-orbit coupling. Both phenomena use the Pauli matrices to represent the spin and orbital angular momentum magnetic interactions. The appropriate Hamiltonian operator is constructed using the Pauli matrices and its eigenvalues and eigenvectors are calculated, and the results interpreted.

http://www.users.csbsju.edu/~frioux/stability/HAtomHyperfineShort.pdf

http://www.users.csbsju.edu/~frioux/stability/DeuteriumHyperfine.pdf

http://www.users.csbsju.edu/~frioux/stability/SpinOrbitTensor.pdf

http://www.users.csbsju.edu/~frioux/stability/SingletTripletStates.pdf

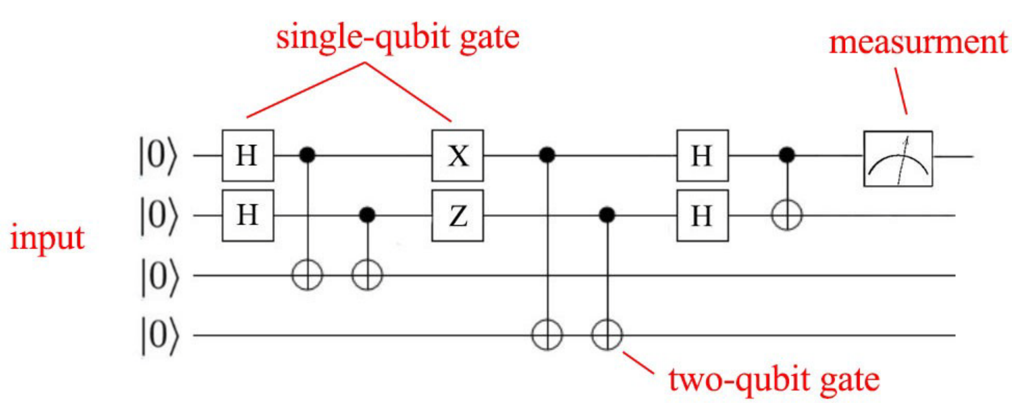
In nuclear magnetic resonance (NMR) proton spins interact with each other and with an external magnetic field. Because of the interaction with the external magnetic field NMR is a little more complicated than the hyperfine interaction and spin-orbit coupling studied in the previous section.

http://www.users.csbsju.edu/~frioux/nmr/DibromothiopheneNMR.pdf

http://www.users.csbsju.edu/~frioux/nmr/ABC-NMR-Tensor.pdf

http://www.users.csbsju.edu/~frioux/nmr/HD-NMR.pdf

Now we move to quantum computing and quantum information theory. These areas make use of quantum circuits like the one shown below. This is a four qubit circuit that employs the Pauli X and Z matrices and three different controlled not (X) gates. The Hadamard gate, H, is an even superposition of the X and Z matrices.



When the Pauli matrices appear in a quantum computer circuit they are called gates. The following two-qubit quantum circuit provides a simple introduction to parallel quantum computation.

http://www.users.csbsju.edu/~frioux/q-intro/QuantumComputerIntro.pdf

The next tutorial uses a three qubit circuit to carry out a more complicated calculation. It also provides a detailed analysis of the quantum circuit operation.

http://www.users.csbsju.edu/~frioux/q-intro/QuantumParallelComputation.pdf

More quantum computer applications are available at:

http://www.users.csbsju.edu/~frioux/workinprogress.html#QCompute

Among the most intriguing quantum phenomenon is teleportation, which uses entanglement and a classical communication channel to transfer a quantum state from one location to another.

http://www.users.csbsju.edu/~frioux/q-intro/BriefTeleportationSummary.pdf

Additional examples of teleportation can be found at:

http://www.users.csbsju.edu/~frioux/workinprogress.html#Teleport

The following tutorials illustrate quantum data base searching and superdense coding methods, important applications in quantum information theory.

http://www.users.csbsju.edu/~frioux/q-intro/GroverMonroe.pdf

http://www.users.csbsju.edu/~frioux/q-intro/QuantumDenseCoding.pdf

We end this short course with the conflict between quantum mechanics and the classical view of reality held by Einstein and others known as local realism. Until the work of John Bell this battle was regarded by most scientists as a distracting philosophical debate. In 1964 Bell demonstrated that experiments were possible for which quantum theory and local realism gave conflicting predictions, thus moving the disagreement from the realm of philosophical debate to the jurisdiction of the laboratory. In what follows violations of Bell's theorem are illustrated using both spin and photon systems. So naturally the Pauli operators play an important role.

In 1981 Richard Feynman gave a lecture titled "Simulating Nature with Computers." His thesis was that the simulation of nature at the nanoscopic level required a quantum computer. "I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit. And if you want to make a simulation of nature, you'd better make it quantum mechanical …" The following tutorials analyze experiments proposed by Bohm, Mermin, GHZ and Feynman, which are simulated using quantum computer circuits.

http://www.users.csbsju.edu/~frioux/q-intro/BohmEPR-Extended.pdf

http://www.users.csbsju.edu/~frioux/q-intro/EPRBell-Revised.pdf

http://www.users.csbsju.edu/~frioux/q-intro/GHZ-Brief-Simulation.pdf

http://www.users.csbsju.edu/~frioux/q-intro/NewFeynman6.pdf

Additional examples of the battle between quantum mechanics and local realism are available at:

http://www.users.csbsju.edu/~frioux/workinprogress.html#Bell