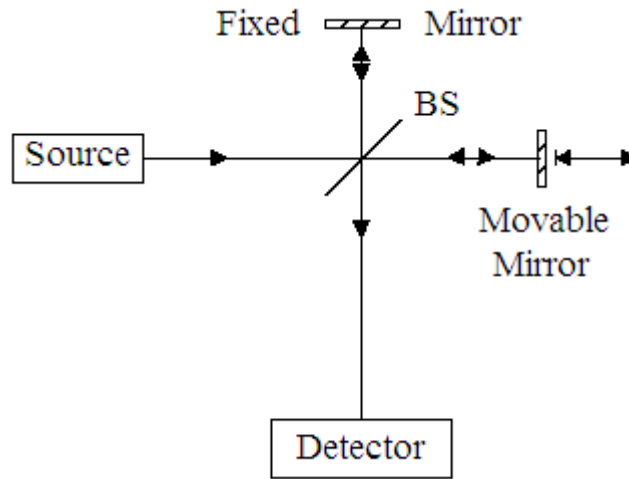


Two Analyses of the Michelson Interferometer

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Path difference between interferometer arms is δ . Phase accumulated due to path difference: $\exp\left(i \cdot \frac{2 \cdot \pi \cdot \delta}{\lambda}\right)$

S stands for source, **D** for detector, **T** for transmitted and **R** for reflected. The evolution of the photon wave function at various stages is given below.

$$S = \frac{1}{\sqrt{2}} \cdot (T + iR) \quad T = \frac{\exp\left(i \cdot \frac{2 \cdot \pi \cdot \delta}{\lambda}\right)}{\sqrt{2}} \cdot (i \cdot D + S) \quad R = \frac{1}{\sqrt{2}} \cdot (D + i \cdot S)$$

$$S = \frac{1}{\sqrt{2}} \cdot (T + iR) \quad \left\{ \begin{array}{l} \text{substitute, } T = \frac{\exp\left(i \cdot \frac{2 \cdot \pi \cdot \delta}{\lambda}\right)}{\sqrt{2}} \cdot (i \cdot D + S) \\ \text{substitute, } R = \frac{1}{\sqrt{2}} \cdot (D + i \cdot S) \end{array} \right. \rightarrow S = -\frac{S}{2} + \frac{S \cdot e^{\frac{2i \cdot \pi \cdot \delta}{\lambda}}}{2} + \frac{D \cdot e^{\frac{2i \cdot \pi \cdot \delta}{\lambda}} \cdot i}{2} + \frac{D \cdot i}{2}$$

The probability the photon will arrive at the detector is the square of the absolute magnitude of the coefficient of **D**.

$$\frac{-1}{2} \cdot i \cdot \left(e^{-2i \cdot \pi \cdot \frac{\delta}{\lambda}} + 1 \right) \cdot \frac{1}{2} \cdot i \cdot \left(e^{2i \cdot \pi \cdot \frac{\delta}{\lambda}} + 1 \right) \text{ simplify } \rightarrow \cos\left(\frac{\pi \cdot \delta}{\lambda}\right)^2$$

The probability the photon will be returned to the source is the square of the absolute magnitude of the coefficient of **S**.

$$\frac{1}{2} \cdot \left(e^{\frac{-2i \cdot \pi \cdot \delta}{\lambda}} - 1 \right) \cdot \frac{1}{2} \cdot \left(e^{\frac{2i \cdot \pi \cdot \delta}{\lambda}} - 1 \right) \text{ simplify } \rightarrow \sin\left(\frac{\pi \cdot \delta}{\lambda}\right)^2$$

The same results are now illustrated using a matrix mechanics approach. Horizontal and vertical motion of the photon are represented by vectors. The source emits a horizontal photon, the detector receives a vertical photon. The beam splitter and the phase shift due to path length difference are represented by matrices. A matrix representation for the mirrors is unnecessary because they simply return the photon to the beam splitter.

$$\begin{array}{ll} \text{Horizontal motion:} & \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \text{Vertical motion:} & \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ \text{Beam splitter:} & \text{BS} := \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \\ \text{Phase shift:} & \text{A}(\delta) := \begin{pmatrix} e^{2i\pi\frac{\delta}{\lambda}} & 0 \\ 0 & 1 \end{pmatrix} \end{array}$$

Calculate the probability amplitude and probability that the photon will arrive at the detector.

$$\begin{aligned} (0 \ 1) \cdot \text{BS} \cdot \begin{pmatrix} e^{2i\pi\frac{\delta}{\lambda}} & 0 \\ 0 & 1 \end{pmatrix} \cdot \text{BS} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} &\rightarrow \frac{e^{2i\pi\frac{\delta}{\lambda}} \cdot i}{2} + \frac{1}{2} \cdot i \\ \left(\frac{-2i\pi\delta}{\lambda} \cdot -i + \frac{1}{2} \cdot -i \right) \cdot \left(\frac{2i\pi\delta}{\lambda} \cdot i + \frac{1}{2} \cdot i \right) &\text{simplify} \rightarrow \cos^2\left(\frac{\pi\delta}{\lambda}\right) \end{aligned}$$

Calculate the probability amplitude and probability that the photon will be returned to the source.

$$\begin{aligned} (1 \ 0) \cdot \text{BS} \cdot \begin{pmatrix} e^{2i\pi\frac{\delta}{\lambda}} & 0 \\ 0 & 1 \end{pmatrix} \cdot \text{BS} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} &\rightarrow \frac{e^{2i\pi\frac{\delta}{\lambda}}}{2} - \frac{1}{2} \\ \left(\frac{-2i\pi\delta}{\lambda} - \frac{1}{2} \right) \cdot \left(\frac{2i\pi\delta}{\lambda} - \frac{1}{2} \right) &\text{simplify} \rightarrow \sin^2\left(\frac{\pi\delta}{\lambda}\right) \end{aligned}$$

Plotting these results in units of λ yields: $\delta := 0, .01 \dots 1$

