Matrix Mechanics Analysis of Quantum Phenomena

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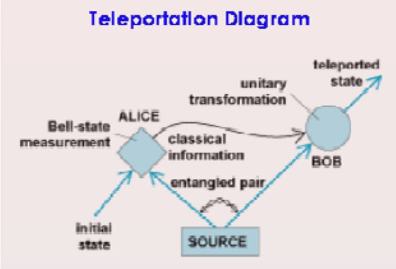
Introduction

Matrix mechanics calculations are a useful tool for illustrating quantum phenomena. This project looked at multiple applications such as quantum computing, quantum teleportation, quantum information theory, and the conflict between quantum theory and classical theories based on local realism. This poster focuses on one of these applications, quantum teleportation. Here we see how simple matrix manipulations can transport an initial state from one location to another, while preserving the state's integrity.

Quantum Teleportation

The following three wire quantum circuit teleports the state at the left on the upper wire to the lower wire at the right. The lower wires on the left initially share an entangled Bell state.

	Initial	1	2	3	4		Final
Teleportee	$\lvert\Psi\rangle = \begin{pmatrix}\sqrt{1/3}\\ \sqrt{2/3}\end{pmatrix}$	•	H	⋯	•	▷	$\frac{1}{\sqrt{2}}(\lvert 0\rangle + \lvert 1\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$
	•	⊕	⋯	•	|	▷	$\frac{1}{\sqrt{2}}(\lvert 0\rangle + \lvert 1\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$
Bell State $\sqrt{1/2}(1\ \ 0\ \ 0\ \ 1)^T$	•	⋯	⋯	⊕	Z	▷	$\lvert\Psi\rangle = \begin{pmatrix}\sqrt{1/3}\\ \sqrt{2/3}\end{pmatrix} = \begin{pmatrix}0.577\\0.816\end{pmatrix}$



Circuit Gates in Matrix Format

$$I := \begin{pmatrix}1&0\\0&1\end{pmatrix} \quad H := \frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix} \quad Z := \begin{pmatrix}1&0\\0&-1\end{pmatrix} \quad CNOT := \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$$

$$CnZ := \begin{pmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&-1\end{pmatrix}$$

Initial state:

$$\begin{pmatrix}\sqrt{\frac{1}{3}}\\ \sqrt{\frac{2}{3}}\end{pmatrix} \cdot \sqrt{\frac{1}{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\right] = \begin{pmatrix}\sqrt{\frac{1}{3}}\\ \sqrt{\frac{2}{3}}\end{pmatrix} \cdot \sqrt{\frac{1}{2}} \cdot \begin{pmatrix}1\\0\\0\\1\end{pmatrix}$$

$$\Psi_i := \sqrt{\frac{1}{2}} \cdot \begin{pmatrix}\sqrt{\frac{1}{3}}\\0\\0\\ \sqrt{\frac{1}{3}}\\ \sqrt{\frac{2}{3}}\\0\\0\\ \sqrt{\frac{2}{3}}\end{pmatrix} \qquad \Psi_i = \begin{pmatrix}0.408\\0\\0\\0.408\\0.577\\0\\0\\0.577\end{pmatrix}$$

Teleportation Circuit

Step1 := kronecker(CNOT, I) Step2 := kronecker(H, kronecker(I, I)) Step3 := kronecker(I, CNOT) Step4 := CnZ

TC := Step4·Step3·Step2·Step1

$$TC = \sqrt{\frac{1}{2}} \cdot \begin{pmatrix}1&0&0&0&0&0&1&0\\0&1&0&0&0&0&0&1\\0&0&0&1&0&1&0&0\\0&0&1&0&1&0&0&0\\1&0&0&0&0&0&-1&0\\0&-1&0&0&0&0&0&1\\0&0&0&1&0&-1&0&0\\0&0&-1&0&1&0&0&0\end{pmatrix}$$

Final state:

$$\Psi_f := TC \cdot \Psi_i = \begin{pmatrix}0.289\\0.408\\0.289\\0.408\\0.289\\0.408\\0.289\\0.408\end{pmatrix} \qquad \Psi_f = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \cdot \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \cdot \begin{pmatrix}\sqrt{\frac{1}{3}}\\ \sqrt{\frac{2}{3}}\end{pmatrix}$$

Summary

This example illustrates the role of the superposition principle and entanglement in the transfer of the state of a qubit (quantum bit, teleportee) from one place to another. The graphical representation of the teleportation protocol shown at the upper right is implemented by the quantum circuit immediately to its left, where steps 3 and 4 replace the classical communication channel between Alice and Bob. In teleportation and the other applications mentioned in the Introduction, entanglement provides a quantum channel for the transfer of information.