A Quantum Circuit for a Mach-Zehnder Interferometer

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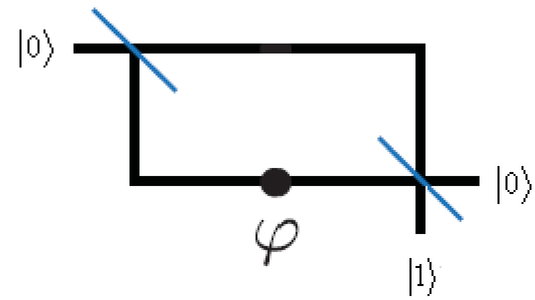
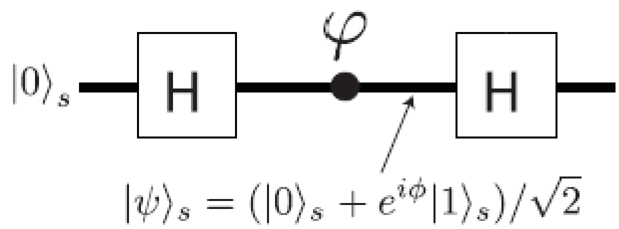
Schematic diagram of a Mach-Zehnder interferometer (MZI).



The following quantum circuit simulates the MZI.



The arms of the MZI are represented by the following orthonormal basis. $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

The matrices representing the Hadamard and phase shift gates are: $H := \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ $\quad A(\phi) := \begin{pmatrix} 1 & 0 \\ 0 & e^{i \cdot \phi} \end{pmatrix}$

Step-by-step through the circuit. The first Hadamard gate creates a superposition of the |0 > and |1 > states. The phase shifter operates on the lower arm of the MZI. The final Hadamard gate allows interference between the two arms of the MZI.

$$H \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix} \qquad A(\phi) \cdot H \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2} \cdot e^{\phi \cdot i}}{2} \end{pmatrix} \qquad H \cdot A(\phi) \cdot H \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{e^{\phi \cdot i}}{2} + \frac{1}{2} \\ \frac{1}{2} - \frac{e^{\phi \cdot i}}{2} \end{pmatrix}$$

Probability of detection at the |0 > port:

$$P_0(\phi) := \left[\left| \begin{pmatrix} 1 & 0 \end{pmatrix} \cdot H \cdot A(\phi) \cdot H \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right| \right]^2 \;\Big|\; \begin{matrix} \text{assume}, \phi = \text{real} \\ \text{simplify} \end{matrix} \;\rightarrow \frac{\cos(\phi)}{2} + \frac{1}{2}$$

Probability of detection at the |1 > port:

$$P_1(\phi) := \left[\left| \begin{pmatrix} 0 & 1 \end{pmatrix} \cdot H \cdot A(\phi) \cdot H \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right| \right]^2 \;\Big|\; \begin{matrix} \text{assume}, \phi = \text{real} \\ \text{simplify} \end{matrix} \;\rightarrow \frac{1}{2} - \frac{\cos(\phi)}{2}$$

A graphical representation of the above calculations shows the interference effects as a function of ϕ.

$$\phi := 0, .01\cdot\pi .. 4\cdot\pi$$