Quantum v. Realism

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This tutorial demonstrates the conflict between quantum theory and realism in an experiment described in *Physical Review Letters* on October 31, 2003 by Zhao, *et al*. titled "Experimental Violation of Local Realism by Four-Photon GHZ Entanglement." It draws on the methodology outlined by N. David Mermin in two articles in the general physics literature: *Physics Today*, June 1990; *American Journal of Physics*, August 1990.

The experiment involves the measurement of the diagonal and circular polarization states of a four-photon entangled state using the following measurement protocols.

$$\sigma_d^1 \otimes \sigma_d^2 \otimes \sigma_d^3 \otimes \sigma_d^4 \qquad \sigma_d^1 \otimes \sigma_c^2 \otimes \sigma_d^3 \otimes \sigma_c^4 \qquad \sigma_d^1 \otimes \sigma_d^2 \otimes \sigma_c^3 \otimes \sigma_c^4 \qquad \sigma_d^1 \otimes \sigma_c^2 \otimes \sigma_c^3 \otimes \sigma_d^4$$

The individual polarization operators and their eigenvalues are:

$$D := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \text{eigenvals}(D) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad C := \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \text{eigenvals}(C) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

The composite operators are formed by tensor matrix multiplication, where ***kronecker*** is Mathcad's command for tensor multiplication.

$$\sigma_{dddd} := \text{kronecker}(D, \text{kronecker}(D, \text{kronecker}(D, D))) \qquad \sigma_{dcdc} := \text{kronecker}(D, \text{kronecker}(C, \text{kronecker}(D, C)))$$

$$\sigma_{ddcc} := \text{kronecker}(D, \text{kronecker}(D, \text{kronecker}(C, C))) \qquad \sigma_{dccd} := \text{kronecker}(D, \text{kronecker}(C, \text{kronecker}(C, D)))$$

These operators commute with each other allowing them to have simultaneous eigenvalues.

$$\sigma_{dddd} \cdot \sigma_{dcdc} - \sigma_{dcdc} \cdot \sigma_{dddd} \rightarrow 0 \qquad \sigma_{dddd} \cdot \sigma_{dccd} - \sigma_{dccd} \cdot \sigma_{dddd} \rightarrow 0 \qquad \sigma_{dddd} \cdot \sigma_{ddcc} - \sigma_{ddcc} \cdot \sigma_{dddd} \rightarrow 0$$

$$\sigma_{ddcc} \cdot \sigma_{dcdc} - \sigma_{dcdc} \cdot \sigma_{ddcc} \rightarrow 0 \qquad \sigma_{ddcc} \cdot \sigma_{dccd} - \sigma_{dccd} \cdot \sigma_{ddcc} \rightarrow 0 \qquad \sigma_{dcdc} \cdot \sigma_{dccd} - \sigma_{dccd} \cdot \sigma_{dcdc} \rightarrow 0$$

Next we show that the null matrix results when the fourth operator is added to the product of the first three.

$$\sigma_{dddd} \cdot \sigma_{dcdc} \cdot \sigma_{ddcc} + \sigma_{dccd} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

To facilitate further analysis, the null result is written as follows.

$$\sigma_{dddd}\cdot\sigma_{dcdc}\cdot\sigma_{ddcc} = -\sigma_{dccd}$$

Local realism maintains that objects have values for observable properties that exist prior to measurement and are independent of the context of the measurement choice (noncontextual). If this assumption is valid, then the operators highlighted with the same color must have the same eigenvalues (+1 or -1) and therefore the product of their eigenvalues must be unity.

$$\left(\sigma_d^1 \otimes \sigma_d^2 \otimes \sigma_d^3 \otimes \sigma_d^4\right)\left(\sigma_d^1 \otimes \sigma_c^2 \otimes \sigma_d^3 \otimes \sigma_c^4\right)\left(\sigma_d^1 \otimes \sigma_d^2 \otimes \sigma_c^3 \otimes \sigma_c^4\right) = -\left(\sigma_d^1 \otimes \sigma_c^2 \otimes \sigma_c^3 \otimes \sigma_d^4\right)$$

Thus, applying a classical concept (noncontextual realism) to the above quantum mechanical equation leads to the following contradiction.

$$\sigma_d^1 \otimes \sigma_c^2 \otimes \sigma_c^3 \otimes \sigma_d^4 = -\sigma_d^1 \otimes \sigma_c^2 \otimes \sigma_c^3 \otimes \sigma_d^4$$

The experimental results reported by Zhao, *et al*. validate the quantum mechanical analysis, and contradict the realistic interpretation. See the preceding tutorial for a summary of their experimental results. Although it wasn't required, here's the four-photon state vector and calculations using it that are consistent with previous analysis.

$$\Psi := \frac{1}{\sqrt{2}}\cdot\begin{pmatrix}0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\end{pmatrix}^T$$

$$\sigma_{dddd}\cdot\sigma_{dcdc}\cdot\sigma_{ddcc} = -\sigma_{dccd}$$

$$\Psi^T\cdot\sigma_{dddd}\cdot\sigma_{dcdc}\cdot\sigma_{ddcc}\cdot\Psi = 1 \qquad \Psi^T\cdot\sigma_{dccd}\cdot\Psi = -1$$

A Quirk Quantum Simulator (algassert.com/quirk) circuit for calculating these expectation values is sketched below. X and Y are the Pauli operators which were labelled d and c above.

```
|0⟩  H  •              X    X    X    X        •  H      |0⟩
        |                                      |
|1⟩     ⊕  •           X    Y    X    Y     •  ⊕         |1⟩
           |              or   or   or       |         ±
|0⟩        ⊕  •        X    X    Y    Y  •   ⊕            |0⟩
              |                          |
|1⟩           ⊕        X    Y    Y    X  ⊕                |1⟩
```