A Summary of Feynman's "Simulating Physics with Computers"

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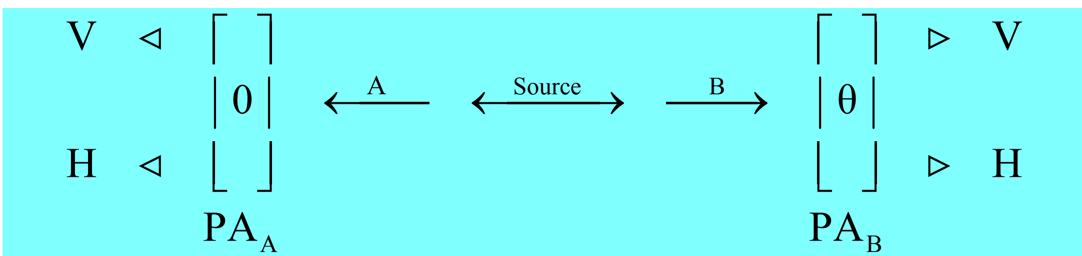
This tutorial is based on "Simulating Physics with Computers" by Richard Feynman, published in the *International Journal of Theoretical Physics* (volume 21, pages 481-485), and Julian Brown's *Quest for the Quantum Computer* (pages 91-100). Feynman used the experiment outlined below to establish that a local classical computer could not simulate quantum physics.

A two-stage atomic cascade emits entangled photons (A and B) in opposite directions with the same circular polarization according to observers in their path.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[|L\rangle_A|L\rangle_B + |R\rangle_A|R\rangle_B\right]$$

The experiment involves the measurement of photon polarization states in the vertical/horizontal measurement basis, and allows for the rotation of the right-hand detector through an angle of θ, in order to explore the consequences of quantum mechanical entanglement. PA stands for polarization analyzer and could simply be a calcite crystal.



In vector notation the left- and right-circular polarization states are expressed as follows:

Left circular polarization: $L := \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 \\ i \end{pmatrix}$ Right circular polarization: $R := \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 \\ -i \end{pmatrix}$

In tensor notation the initial photon state is,

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[|L\rangle_A|L\rangle_B + |R\rangle_A|R\rangle_B\right] = \frac{1}{2\sqrt{2}}\left[\begin{pmatrix} 1 \\ i \end{pmatrix}_A \otimes \begin{pmatrix} 1 \\ i \end{pmatrix}_B + \begin{pmatrix} 1 \\ -i \end{pmatrix}_A \otimes \begin{pmatrix} 1 \\ -i \end{pmatrix}_B\right] = \frac{1}{2\sqrt{2}}\left[\begin{pmatrix} 1 \\ i \\ i \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ -i \\ -i \\ -1 \end{pmatrix}\right] = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

However, as mentioned above, the photon polarization measurements will actually be made in the vertical/horizontal basis. These polarization states in vector representation are:

Vertical polarization: $V := \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ Horizontal polarization: $H := \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

It is easy to show that the equivalent vertical/horizontal polarization state is,

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[|V\rangle_A |V\rangle_B - |H\rangle_A |H\rangle_B\right] = \frac{1}{2\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}_A \otimes \begin{pmatrix}1\\0\end{pmatrix}_B - \begin{pmatrix}0\\1\end{pmatrix}_A \otimes \begin{pmatrix}0\\1\end{pmatrix}_B\right] = \frac{1}{2\sqrt{2}}\left[\begin{pmatrix}1\\0\\0\\0\end{pmatrix} - \begin{pmatrix}0\\0\\0\\1\end{pmatrix}\right] = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}$$

There are four measurement outcomes: both photons are vertically polarized, both are horizontally polarized, one is vertical and the other horizontal, and vice versa. The tensor representation of these measurement states are provided below.

$$|VV\rangle = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad |VH\rangle = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \quad |HV\rangle = \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \quad |HH\rangle = \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

$$\Psi := \frac{1}{\sqrt{2}} \cdot \begin{pmatrix}1\\0\\0\\-1\end{pmatrix} \qquad \mathrm{VV} := \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \qquad \mathrm{VH} := \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \qquad \mathrm{HV} := \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \qquad \mathrm{HH} := \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

Next, the operator representing the rotation of PA_B by an angle θ relative to PA_A is constructed using matrix tensor multiplication. **Kronecker** is Mathcad's command for tensor matrix multiplication.

$$\mathrm{RotOp}(\theta) := \mathrm{kronecker}\left[\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}, \begin{pmatrix}\cos(\theta) & \sin(\theta)\\-\sin(\theta) & \cos(\theta)\end{pmatrix}\right]$$

Now we can get on with some actual calculations. If the relative angle between PA_A and PA_B is zero degrees we observe perfect correlation. In other words, 50% of the time the analyzers agree that both photons are vertically polarized, and 50% of the time they agree that they are horizontally polarized.

Perfect correlation: $\theta := 0 \cdot \mathrm{deg}$

$$\begin{bmatrix}\left(\mathrm{VV}^T \cdot \mathrm{RotOp}(\theta) \cdot \Psi\right)^2 & \left(\mathrm{VH}^T \cdot \mathrm{RotOp}(\theta) \cdot \Psi\right)^2\\ \left(\mathrm{HV}^T \cdot \mathrm{RotOp}(\theta) \cdot \Psi\right)^2 & \left(\mathrm{HH}^T \cdot \mathrm{RotOp}(\theta) \cdot \Psi\right)^2\end{bmatrix} = \begin{pmatrix}50 & 0\\0 & 50\end{pmatrix}\%$$

However, if the relative angle between the two polarization analyzers is 90 degrees, perfect anti-correlation is observed - the photons always are detected with the opposite polarizations.

Perfect anti-correlation: $\theta := 90 \cdot \mathrm{deg}$

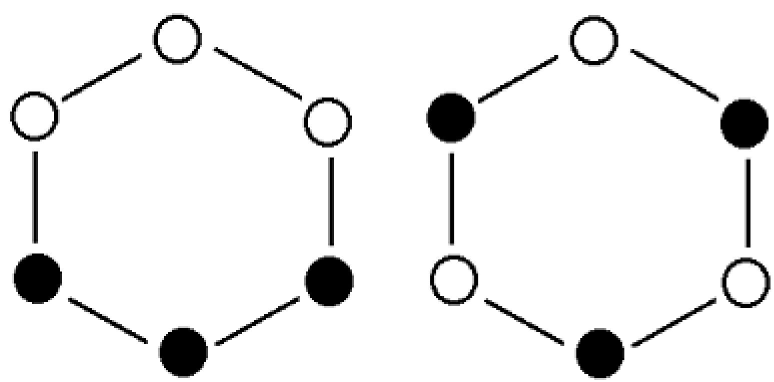
$$\begin{bmatrix}\left(\mathrm{VV}^T \cdot \mathrm{RotOp}(\theta) \cdot \Psi\right)^2 & \left(\mathrm{VH}^T \cdot \mathrm{RotOp}(\theta) \cdot \Psi\right)^2\\ \left(\mathrm{HV}^T \cdot \mathrm{RotOp}(\theta) \cdot \Psi\right)^2 & \left(\mathrm{HH}^T \cdot \mathrm{RotOp}(\theta) \cdot \Psi\right)^2\end{bmatrix} = \begin{pmatrix}0 & 50\\50 & 0\end{pmatrix}\%$$

By comparison, if the relative angle between the analyzers is 30 degrees, the analysers behave the same way 75% of the time.

$$\theta := 30 \cdot \mathrm{deg} \qquad \begin{bmatrix} \left(VV^T \cdot \mathrm{RotOp}(\theta) \cdot \Psi\right)^2 & \left(VH^T \cdot \mathrm{RotOp}(\theta) \cdot \Psi\right)^2 \\ \left(HV^T \cdot \mathrm{RotOp}(\theta) \cdot \Psi\right)^2 & \left(HH^T \cdot \mathrm{RotOp}(\theta) \cdot \Psi\right)^2 \end{bmatrix} = \begin{pmatrix} 37.5 & 12.5 \\ 12.5 & 37.5 \end{pmatrix} \%$$

These quantum calculations are in agreement with experimental results. As we shall see they cannot be explained by a local realistic hidden-variable model of reality.

If you subscribe to the principle of local realism and believe that objects have well-defined properties independent of measurement or observation, the first two results (θ = 0 degrees, θ = 90 degrees) require that the photons carry the following instruction sets, where the hexagonal vertices refer to θ values of 0, 30, 60, 90, 120, and 150 degrees. There are eight possible instruction sets, six of the type on the left and two of the type on the right. The white circles represent vertical polarization and the black circles represent horizontal polarization. In any given measurement, according to local realism, both photons (A and B) carry identical instruction sets, in other words the same one of the eight possible sets.



The problem is that while these instruction sets are in agreement with the 0 and 90 degree results, they can't explain the 30 degree data. The figure on the left shows that the same result should be obtained 2/3 of the time (4/6) and the figure on the right never. Thus, local realism predicts that the same result should be obtained 50% of the time, as opposed to the actual result of 75% of the time.

$$\frac{6 \cdot \frac{2}{3} + 2 \cdot 0}{8} = 50\%$$

When Feynman gets to this point in "Simulating Physics with Computers" he writes,

> That's all. That's the difficulty. That's why quantum mechanics can't seem to be imitable by a local classical computer.

A local classical computer manipulates bits which are in well-defined states, 0s and 1s, shown above graphically in white and black. However, these classical states are incompatible with the quantum mechanical analysis which **is** consistent with experimental results. This two-photon experiment demonstrates that simulation of quantum physics requires a computer that can manipulate 0s and 1s, superpositions of 0 and 1, and entangled superpositions of 0s and 1s. Simulation of quantum physics requires a quantum computer!

Earlier in his paper Feynman sharpened the focus of his analysis by saying "But the physical world is quantum mechanical, and therefore the proper problem is the simulation of quantum physics ..."

He ends his presentation with the following remark.

> And I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, ...

In conclusion, the quantum mechanical calculations are presented graphically for all θ values between 0 and 180 degrees.

$$P_{same}(\theta) := \left(VV^T \cdot RotOp(\theta) \cdot \Psi\right)^2 + \left(HH^T \cdot RotOp(\theta) \cdot \Psi\right)^2$$

$$P_{diff}(\theta) := \left(VH^T \cdot RotOp(\theta) \cdot \Psi\right)^2 + \left(HV^T \cdot RotOp(\theta) \cdot \Psi\right)^2$$

$$\theta := 0 \cdot deg, 1 \cdot deg .. 180 \cdot deg$$