Elements of Reality

Frank Rioux
Emeritus Professor of Chemistry
CSB | SJU

N. David Mermin published two articles in the physics literature (*Physics Today*, June 1990; *American Journal of Physics*, August 1990) on a Greenberger-Horne-Zeilinger (GHZ) gedanken experiment (see *American Journal of Physics*, December 1990 for a description) involving spins that sharply reveals the clash between local realism and the quantum view of reality.

Three spin-1/2 particles are created in a single event and move apart in the horizontal y-z plane. Subsequent spin measurements will be carried out in units of h/4π using spin operators in the x- and y-directions.

The z-basis eigenfunctions are: $Sz_{up} := \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $Sz_{down} := \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

The x- and y-direction spin operators and eigenvalues: $\sigma_x := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\text{eigenvals}(\sigma_x) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $\sigma_y := \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\text{eigenvals}(\sigma_y) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

The initial spin state for the three spin-1/2 particles in tensor notation is:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}\otimes\begin{pmatrix} 1 \\ 0 \end{pmatrix}\otimes\begin{pmatrix} 1 \\ 0 \end{pmatrix}-\begin{pmatrix} 0 \\ 1 \end{pmatrix}\otimes\begin{pmatrix} 0 \\ 1 \end{pmatrix}\otimes\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]=\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

$$\Psi := \frac{1}{\sqrt{2}}\cdot\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

The following operators represent the measurements to be carried out on spins 1, 2 and 3, in that order.

$$\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3 \qquad \sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3 \qquad \sigma_y^1 \otimes \sigma_y^2 \otimes \sigma_x^3 \qquad \sigma_x^1 \otimes \sigma_x^2 \otimes \sigma_x^3$$

The matrix tensor product is also known as the Kronecker product, which is available in Mathcad. The four operators in tensor format are formed as follows.

$$\sigma_{xyy} := \text{kronecker}(\sigma_x, \text{kronecker}(\sigma_y, \sigma_y)) \qquad \sigma_{yxy} := \text{kronecker}(\sigma_y, \text{kronecker}(\sigma_x, \sigma_y))$$

$$\sigma_{yyx} := \text{kronecker}(\sigma_y, \text{kronecker}(\sigma_y, \sigma_x)) \qquad \sigma_{xxx} := \text{kronecker}(\sigma_x, \text{kronecker}(\sigma_x, \sigma_x))$$

The fact that the operators commute means that they can have simultaneous eigenvalues.

$$\sigma_{xyy}\cdot\sigma_{yxy} - \sigma_{yxy}\cdot\sigma_{xyy} \rightarrow 0 \qquad \sigma_{xyy}\cdot\sigma_{yyx} - \sigma_{yyx}\cdot\sigma_{xyy} \rightarrow 0 \qquad \sigma_{yxy}\cdot\sigma_{yyx} - \sigma_{yyx}\cdot\sigma_{yxy} \rightarrow 0$$

$$\sigma_{xyy}\cdot\sigma_{xxx} - \sigma_{xxx}\cdot\sigma_{xyy} \rightarrow 0 \qquad \sigma_{yxy}\cdot\sigma_{xxx} - \sigma_{xxx}\cdot\sigma_{yxy} \rightarrow 0 \qquad \sigma_{yyx}\cdot\sigma_{xxx} - \sigma_{xxx}\cdot\sigma_{yyx} \rightarrow 0$$

That the initial state is an eigenfunction of these operators with eigenvalues +/-1 is now demonstrated.

$$\Psi^T \cdot \sigma_{xyy} \cdot \Psi = 1 \qquad \Psi^T \cdot \sigma_{yxy} \cdot \Psi = 1 \qquad \Psi^T \cdot \sigma_{yyx} \cdot \Psi = 1 \qquad \Psi^T \cdot \sigma_{xxx} \cdot \Psi = -1$$

Local realism postulates that objects have definite properties prior to and independent of measurement. In this example it assumes that the x- and y-components of the spin are in well-defined states. It will be shown that this assumption leads to a contradiction with these quantum mechanical calculations. It is easy to show that the eight spin measurement instruction sets highlighted below for the x- and y-directions are in agreement with with the first three quantum calculations shown above: xyy, yxy and yyx which each have an eigenvalue of +1.

	1	2	3	4	5	6	7	8
	123	123	123	123	123	123	123	123
x	↑↑↑	↑↓↓	↓↑↓	↓↓↑	↑↓↓	↑↑↑	↓↓↑	↓↑↓
y	↑↑↑	↑↓↓	↓↑↓	↓↓↑	↓↑↑	↓↓↓	↑↑↓	↑↓↑

where $\uparrow = +1 \qquad \downarrow = -1$

However these spin instruction sets, which are the only ones that work for the first three operators, also predict a value of +1 for the $\sigma_x^1\sigma_x^2\sigma_x^3$ operator in stark disagreement with the quantum value of -1. Of course a quantum mechanic has already rejected the instruction sets because σ_x and σ_y don't commute. A quon cannot simultaneously have the well-defined values for spin in the x- and y-directions that the instruction sets show. In other words, if you know the x-row spin states you can't specify the y-row spin states as the table above does.

The local realist is undeterred by this argument and the disagreement with the quantum mechanical calculations, asserting that the fact that quantum theory cannot assign well-defined states to all elements of reality independent of observation is an indication that it provides an incomplete description of reality.

However, results available for experiments of this type with photons support the quantum mechanical predictions and contradict the local realist analysis. A GHZ experiment performed using photons was reported in *Nature* on February 3, 2000.