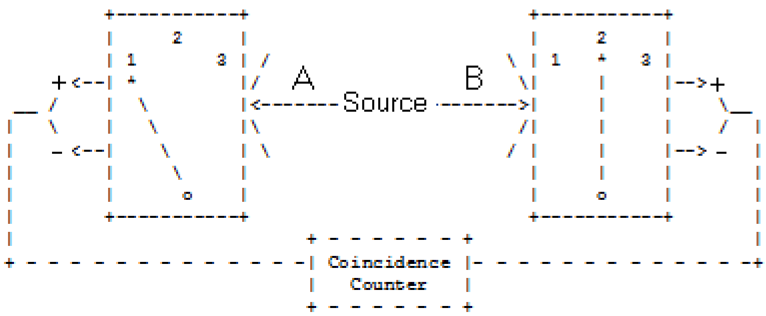
Mermin's EPR Gedanken Experiment

Frank Rioux

In Bohm's EPR thought experiment (*Quantum Theory*, 1951, pp. 614-619), both local realism and quantum mechanics were shown to be consistent with the experimental data. However, the local realistic explanation used composite spin states that were invalid according to quantum theory. The local realist countered that this was an indication that quantum mechanics was incomplete, because it couldn't assign well-defined values to all observable properties. In this example a thought experiment is presented in which the predictions of local realism and quantum mechanics differ. The thought experiment was presented by N. David Mermin in the *American Journal of Physics* (October 1981, pp 941-943) and *Physics Today* (April 1985, pp 38-47).

A spin-1/2 pair is prepared in a singlet state and the individual particles travel in opposite directions to a pair of detectors which are set up to measure spin in three directions in x-z plane: along the z-axis, and angles of 120 and 240 degrees with respect to the z-axis. The detector settings are labeled 1, 2 and 3, respectively.



The switches on the detectors are set randomly so that all nine possible settings of the two detectors occur with equal frequency.

Local realism holds that objects have properties independent of measurement and that measurements at one location on a particle cannot influence measurements of another particle at another distant location even if the particles were created in the same event. Local realism maintains that the spin-1/2 particles carry instruction sets (hidden variables) which dictate the results of subsequent measurements. Prior to measurement the particles are in an unknown but well-defined state.

The following table presents the experimental results expected on the basis of local realism. Singlet spin states have opposite spin values for each of the three measurement directions. For example, if A's spin state is (+-+), then B's spin state is (-+-). If A's detector is set to spin direction "1" and B's detector is set to spin direction "3" the measured result will be recorded as +-.

Spin States	Detector Settings								
	12	13	21	23	31	32	11	22	33
+++/---	+-	+-	+-	+-	+-	+-	+-	+-	+-
++-/--+	+-	++	+-	++	--	--	+-	+-	-+
+-+/-+-	++	+-	--	--	+-	++	+-	-+	+-
-++/+--	--	--	++	+-	++	+-	-+	+-	+-
+--/-++	++	++	--	-+	--	-+	+-	-+	-+
-+-/+-+	--	-+	++	++	-+	--	-+	+-	-+
--+/++-	-+	--	-+	--	++	++	-+	-+	+-
---/+++	-+	-+	-+	-+	-+	-+	-+	-+	-+

Note that there are eight spin states and nine possible detector settings, giving 72 possible measurement outcomes all of which are equally probable.

The table shows that the assumption that the singlet-state particles have well-defined spin states prior to measurement requires that the probability the detectors will register opposite spin values is 0.67 (48/72). If the detectors are set to the same direction, they always register different spin values (24/24), and if they are set to different directions the probability they will register different spin values is 0.50 (24/48). Quantum mechanics disagrees with this local realistic analysis.

The singlet state produced by the source is the following entangled superposition, where the arrows indicate the spin orientation for any direction in the x-z plane. See the Appendix for a proof of this assertion. As noted above the directions used here are 0, 120 and 240 degrees, relative to the z-axis.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\right] = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$$

The orthonormal spin-up and spin-down vectors in the x-z plane are:

$$\varphi_u(\varphi) := \begin{pmatrix} \cos\left(\frac{\varphi}{2}\right) \\ \sin\left(\frac{\varphi}{2}\right) \end{pmatrix} \quad \varphi_d(\varphi) := \begin{pmatrix} -\sin\left(\frac{\varphi}{2}\right) \\ \cos\left(\frac{\varphi}{2}\right) \end{pmatrix} \quad \varphi_u(\varphi)^T \cdot \varphi_u(\varphi) \text{ simplify } \rightarrow 1 \quad \varphi_d(\varphi)^T \cdot \varphi_d(\varphi) \text{ simplify } \rightarrow 1$$

$$\varphi_d(\varphi)^T \cdot \varphi_u(\varphi) \text{ simplify } \rightarrow 0$$

For spin measurements in which the detectors are set at the same angle, quantum mechanics and local realism agree that the detectors will register opposite spins. Therefore we will concentrate on what quantum mechanics predicts when the detectors are set at different angles. Recall that local realism predicts that the detectors will behave differently 50% of the time.

If the particle at detector A is found to be spin-up in the z-direction, it follows from the singlet spin state that the particle at B is spin-down in the z-direction, $\varphi_d(0)$. The probability that it will be detected spin down if detector B is rotated clockwise or counter-clockwise by 120 degrees is,

$$\left(\varphi_d(120\cdot\text{deg})^T \cdot \varphi_d(0\cdot\text{deg})\right)^2 = 0.25 \qquad \left(\varphi_d(240\cdot\text{deg})^T \cdot \varphi_d(0\cdot\text{deg})\right)^2 = 0.25$$

Thus for those cases for which the detectors are set at different angles there is a sharp disagreement between local realism (50%) and quantum mechanics (25%) as to the percentage of the time the detectors behave differently. For all detector settings quantum mechanics predicts that opposite spins will be recorded [(1/3)100% + (2/3)25%] 50% of the time, compared with the 67% calculated on the basis of local realism.

Quantum theory maintains that the discrepancy in the predictions is due to the fact that the local realistic spin states in the left column of the table above are invalid because they assign definite values to incompatible observables. For example, if the z-axis spin value of the first particle is known to be +, then the state of the composite system is not +++/--- or ++-/--+ or +-+/-+- or +--/-++, but +??/-??. The Appendix demonstrates that the spin operators for 0, 120 and 240 degrees do not commute, which means that the observables associated with these operators cannot simultaneously be well defined.

Summary

Local realism maintains that objects have well-defined properties independent of observation, and that the acquisition of a definite value for a property by an object at B due to a measurement carried out on a distant object at A is "spooky action at a distance" and physically unintelligible and therefore impossible.

By contrast quantum theory teaches that quantum particles do not in general have well-defined properties independent of measurement, and that particles with a common origin are in an entangled state and therefore are not independent, no matter how far apart they may be. Together they are in a well-defined correlated state, but their individual properties are uncertain. When measurement determines the state of the particle at A, the correlated property of its distant partner at B becomes known instantaneously.

Appendix

The spin operator in the x-z plane is constructed from the Pauli spin operators in the x- and z-directions. φ is the angle of orientation of the measurement magnet with the z-axis. Note that the Pauli operators measure spin in units of h/4π.

$$\sigma_z := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \sigma_x := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad S(\varphi) := \cos(\varphi)\cdot\sigma_z + \sin(\varphi)\cdot\sigma_x \rightarrow \begin{pmatrix} \cos(\varphi) & \sin(\varphi) \\ \sin(\varphi) & -\cos(\varphi) \end{pmatrix}$$

The following calculations demonstrate that the spin operators do not commute and therefore represent incompatible observables. In other words, they are observables that cannont simulataneously be in well-defined states.

$$S(0\cdot deg)\cdot S(120\cdot deg) - S(120\cdot deg)\cdot S(0\cdot deg) = \begin{pmatrix} 0 & 1.732 \\ -1.732 & 0 \end{pmatrix}$$

$$S(0\cdot deg)\cdot S(240\cdot deg) - S(240\cdot deg)\cdot S(0\cdot deg) = \begin{pmatrix} 0 & -1.732 \\ 1.732 & 0 \end{pmatrix}$$

$$S(120\cdot deg)\cdot S(240\cdot deg) - S(240\cdot deg)\cdot S(120\cdot deg) = \begin{pmatrix} 0 & 1.732 \\ -1.732 & 0 \end{pmatrix}$$

For the singlet state produced by the source the arrows indicate the spin orientation for any direction in the x-z plane.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\right] = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} \cos(\frac{\varphi}{2}) \\ \sin(\frac{\varphi}{2}) \end{pmatrix} \otimes \begin{pmatrix} -\sin(\frac{\varphi}{2}) \\ \cos(\frac{\varphi}{2}) \end{pmatrix} - \begin{pmatrix} -\sin(\frac{\varphi}{2}) \\ \cos(\frac{\varphi}{2}) \end{pmatrix} \otimes \begin{pmatrix} \cos(\frac{\varphi}{2}) \\ \sin(\frac{\varphi}{2}) \end{pmatrix}\right] = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$$