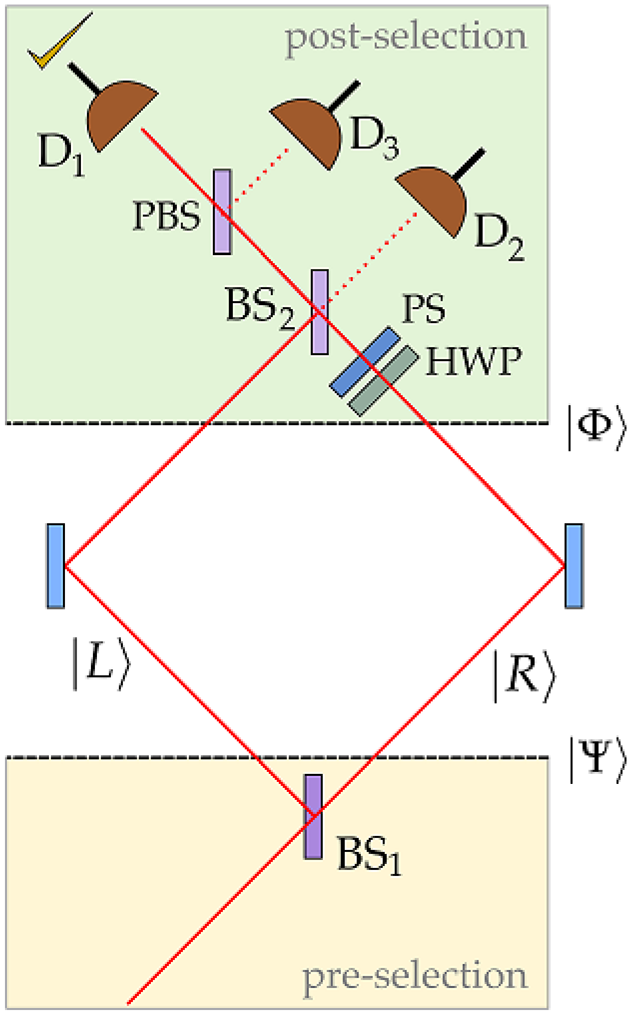
A Quantum Optical Cheshire Cat

Frank Rioux

The following is a summary of "Quantum Cheshire Cats" by Aharonov, Popescu, Rohrlich and Skrzypczyk which was published in the *New Journal of Physics* **15**, 113015 (2013) and can also be accessed at: arXiv:1202.0631v2.



In the absence of the half-wave plate (HWP) and the phase shifter (PS) a horizontally polarized photon entering the interferometer from the lower left (propagating to the upper right) arrives at D_2 with a 90 degree (π/2, i) phase shift. (By convention reflection at a beam splitter introduces a 90 degree phase shift.)

$$|R\rangle|H\rangle \xrightarrow{BS_1} \frac{1}{\sqrt{2}}\left[i|L\rangle+|R\rangle\right]|H\rangle \xrightarrow{BS_2} i|D_2\rangle|H\rangle$$

The state immediately after the first beam splitter is the pre-selected state.

$$|\Psi\rangle=\frac{1}{\sqrt{2}}\left[i|L\rangle+|R\rangle\right]|H\rangle$$

The post-selected state is,

$$|\Phi\rangle=\frac{1}{\sqrt{2}}\left[|L\rangle|H\rangle+|R\rangle|V\rangle\right]$$

The HWP (converts |V> to |H> in the R-branch) and PS transform this state to,

$$|\Phi\rangle \xrightarrow[PS]{HWP} \frac{1}{\sqrt{2}}\left[|L\rangle + i|R\rangle\right]|H\rangle$$

which exits the second beam splitter through the left port to encounter a polarizing beam splitter which transmits horizontal polarization and reflects vertical polarization. Thus, the post-selected state is detected at D_1. The evolution of the post-selected state is summarized as follows:

$$|\Phi\rangle = \frac{1}{\sqrt{2}}\left[|L\rangle|H\rangle + |R\rangle|V\rangle\right] \xrightarrow[PS]{HWP} \frac{1}{\sqrt{2}}\left[|L\rangle + i|R\rangle\right]|H\rangle \xrightarrow{BS_2} i|L\rangle|H\rangle \xrightarrow{PBS} i|D_1\rangle|H\rangle$$

The last term on the right side below is the weak value of **A** multiplied by the probability of its occurrence for the preselected state Ψ and the post-selected state Φ.

$$\langle\Psi|\hat{A}|\Psi\rangle = \sum_j \langle\Psi|\Phi_j\rangle\langle\Phi_j|\hat{A}|\Psi\rangle = \sum_j \langle\Psi|\Phi_j\rangle\langle\Phi_j|\Psi\rangle\frac{\langle\Phi_j|\hat{A}|\Psi\rangle}{\langle\Phi_j|\Psi\rangle} = \sum_j p_j \frac{\langle\Phi_j|\hat{A}|\Psi\rangle}{\langle\Phi_j|\Psi\rangle}$$

The weak value calculations are carried out in a 4-dimensional Hilbert space created by the tensor product of the photon's direction of propagation and polarization vectors.

Direction of propagation vectors: $L := \begin{pmatrix}1\\0\end{pmatrix}$ $R := \begin{pmatrix}0\\1\end{pmatrix}$ Polarization state vectors: $H := \begin{pmatrix}1\\0\end{pmatrix}$ $V := \begin{pmatrix}0\\1\end{pmatrix}$

Pre-selected state: $\Psi = \frac{1}{\sqrt{2}}\cdot(i\cdot L + R)\cdot H = \frac{1}{\sqrt{2}}\cdot\left[i\cdot\begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}\right]\cdot\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\cdot\begin{pmatrix}i\\1\end{pmatrix}\cdot\begin{pmatrix}1\\0\end{pmatrix}$ $\Psi := \frac{1}{\sqrt{2}}\cdot\begin{pmatrix}i\\0\\1\\0\end{pmatrix}$

Post-selected state: $\Phi := \frac{1}{\sqrt{2}}\cdot(L\cdot H + R\cdot V) = \frac{1}{\sqrt{2}}\cdot\left[\begin{pmatrix}1\\0\end{pmatrix}\cdot\begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}\cdot\begin{pmatrix}0\\1\end{pmatrix}\right]$ $\Phi := \frac{1}{\sqrt{2}}\cdot\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$

Direction of propagation operators: $\text{Left} := \begin{pmatrix}1\\0\end{pmatrix}\cdot\begin{pmatrix}1 & 0\end{pmatrix} \rightarrow \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$ $\text{Right} := \begin{pmatrix}0\\1\end{pmatrix}\cdot\begin{pmatrix}0 & 1\end{pmatrix} \rightarrow \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$

Photon angular momentum operator: $\text{Pang} := \begin{pmatrix}0 & -i\\i & 0\end{pmatrix}$ Identity operator: $I := \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

The following weak value calculations show that for the pre- and post-selection ensemble of observations the photon is in the left arm of the interferometer while its angular momentum is in the right arm. Like the case of the Cheshire cat, a photon property has been separated from the photon.

$$\begin{pmatrix} \text{""} & \text{"Left Arm"} & \text{"Right Arm"} \\ \text{"Arm"} & \dfrac{\Phi^T\cdot\text{kronecker}(\text{Left},I)\cdot\Psi}{\Phi^T\cdot\Psi} & \dfrac{\Phi^T\cdot\text{kronecker}(\text{Right},I)\cdot\Psi}{\Phi^T\cdot\Psi} \\ \text{"Pang"} & \dfrac{\Phi^T\cdot\text{kronecker}(\text{Left},\text{Pang})\cdot\Psi}{\Phi^T\cdot\Psi} & \dfrac{\Phi^T\cdot\text{kronecker}(\text{Right},\text{Pang})\cdot\Psi}{\Phi^T\cdot\Psi} \end{pmatrix} = \begin{pmatrix} \text{""} & \text{"Left Arm"} & \text{"Right Arm"} \\ \text{"Arm"} & 1 & 0 \\ \text{"Pang"} & 0 & 1 \end{pmatrix}$$

The following shows the evolution of the pre-selected state to the final state at the detectors. The intermediate is the state illuminating BS_2. The polarization state at the detectors is ignored.

$$|\Psi\rangle \rightarrow \frac{i}{\sqrt{2}}\left[|L\rangle|H\rangle + |R\rangle|V\rangle\right] \rightarrow -\frac{1}{2}|D_1\rangle + \frac{i}{2}|D_3\rangle + \frac{(i-1)}{2}|D_2\rangle$$

Squaring the magnitude of the probability amplitudes shows that the probabilities that D_1, D_3 and D_2 will fire are 1/4, 1/4 and 1/2, respectively. The probability at D_1 is consistent with the probability that the post-selected state is contained in the pre-selected state. A photon in the post-selected state has a probability of 1 of reaching D_1 and it represents a 25% contribution to the pre-selected state.

$$\left(\left|\Phi^{T}\cdot\Psi\right|\right)^{2} \rightarrow \frac{1}{4}$$

Note that the expectation values for the pre-selected state show no path-polarization separation.

$$\begin{bmatrix} \text{""} & \text{"Left Arm"} & \text{"Right Arm"} \\ \text{"Arm"} & \left(\overline{\Psi}\right)^{T}\cdot\text{kronecker}(\text{Left},\text{I})\cdot\Psi & \left(\overline{\Psi}\right)^{T}\cdot\text{kronecker}(\text{Right},\text{I})\cdot\Psi \\ \text{"Pang"} & \left(\overline{\Psi}\right)^{T}\cdot\text{kronecker}(\text{Left},\text{Pang})\cdot\Psi & \left(\overline{\Psi}\right)^{T}\cdot\text{kronecker}(\text{Right},\text{Pang})\cdot\Psi \\ \text{"Hop"} & \left(\overline{\Psi}\right)^{T}\cdot\text{kronecker}\left(\text{Left},\text{H}\cdot\text{H}^{T}\right)\cdot\Psi & \left(\overline{\Psi}\right)^{T}\cdot\text{kronecker}\left(\text{Right},\text{H}\cdot\text{H}^{T}\right)\cdot\Psi \\ \text{"Vop"} & \left(\overline{\Psi}\right)^{T}\cdot\text{kronecker}\left(\text{Left},\text{V}\cdot\text{V}^{T}\right)\cdot\Psi & \left(\overline{\Psi}\right)^{T}\cdot\text{kronecker}\left(\text{Right},\text{V}\cdot\text{V}^{T}\right)\cdot\Psi \end{bmatrix} = \begin{pmatrix} \text{""} & \text{"Left Arm"} & \text{"Right Arm"} \\ \text{"Arm"} & 0.5 & 0.5 \\ \text{"Pang"} & 0 & 0 \\ \text{"Hop"} & 0.5 & 0.5 \\ \text{"Vop"} & 0 & 0 \end{pmatrix}$$

In addition the following table shows that linear polarization (HV) is not separated from the photon's path.

$$\text{HV} := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \begin{pmatrix} \text{""} & \text{"Left Arm"} & \text{"Right Arm"} \\ \text{"Arm"} & \dfrac{\Phi^{T}\cdot\text{kronecker}(\text{Left},\text{I})\cdot\Psi}{\Phi^{T}\cdot\Psi} & \dfrac{\Phi^{T}\cdot\text{kronecker}(\text{Right},\text{I})\cdot\Psi}{\Phi^{T}\cdot\Psi} \\ \text{"HV"} & \dfrac{\Phi^{T}\cdot\text{kronecker}(\text{Left},\text{HV})\cdot\Psi}{\Phi^{T}\cdot\Psi} & \dfrac{\Phi^{T}\cdot\text{kronecker}(\text{Right},\text{HV})\cdot\Psi}{\Phi^{T}\cdot\Psi} \end{pmatrix} = \begin{pmatrix} \text{""} & \text{"Left Arm"} & \text{"Right Arm"} \\ \text{"Arm"} & 1 & 0 \\ \text{"HV"} & 1 & 0 \end{pmatrix}$$

The "Complete Quantum Cheshire Cat" by Guryanova, Brunner and Popescu (arXiv 1203.4215) provides an optical set-up which achieves complete path-polarization separation for the photon.