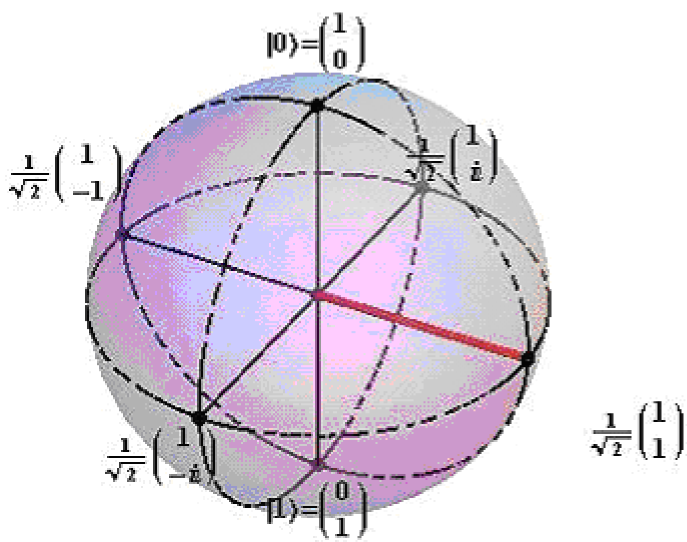
# The Bloch Sphere and the Pauli Matrices

Frank Rioux
Professor Emeritus of Chemistry
College of Saint Benedict|Saint John's University
St. Joseph, MN 56374

According to Wikipedia "the Bloch sphere is a geometrical representation of the state space of a two-level quantum mechanical system...", in this case a spin 1/2 system. The spin up components have eigenvalue +1 and the spin down components have eigenvalue -1.

$$Z_u := \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad Z_d := \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad X_u := \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad X_d := \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad Y_u := \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 \\ i \end{pmatrix} \quad Y_d := \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 \\ -i \end{pmatrix}$$




This figure is from http://demonstrations.wolfram.com/QubitsOnThePoincareBlochSphere/ a contribution by Rudolf Muradian.

With this information the Pauli operators can be calculated and the completeness relation confirmed using the sums of outer products.

$$\widehat{Op} = |u\rangle\langle u| - |d\rangle\langle d| \qquad \widehat{Id} = |u\rangle\langle u| + |d\rangle\langle d|$$

| Pauli Operators | Completeness Relation |
|---|---|
| $\sigma_x := X_u \cdot X_u^T - X_d \cdot X_d^T = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ | $X_u \cdot X_u^T + X_d \cdot X_d^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ |
| $\sigma_y := Y_u \cdot \left(\overline{Y_u}\right)^T - Y_d \cdot \left(\overline{Y_d}\right)^T = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ | $Y_u \cdot \left(\overline{Y_u}\right)^T + Y_d \cdot \left(\overline{Y_d}\right)^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ |
| $\sigma_z := Z_u \cdot Z_u^T - Z_d \cdot Z_d^T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ | $Z_u \cdot Z_u^T + Z_d \cdot Z_d^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ |