Aspect's Experiment in Brief

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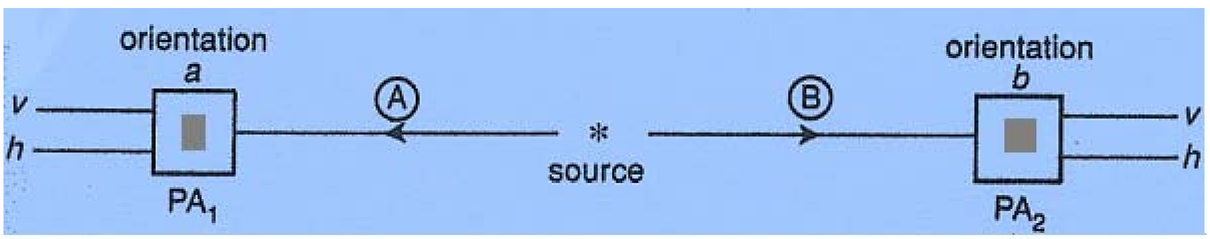
The purpose of this tutorial is restricted to a brief computational summary of the EPR experiment reported by Aspect, Grangier and Roger, "Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedanken Experiment: A New Violation of Bell's Inequalities," in *Phys. Rev. Lett.* **49**, 91 (1982). See Chapter 6 of *The Quantum Challenge* by Greenstein and Zajonc, Chapter 4 of Jim Baggott's *The Meaning of Quantum Theory*, and Chapter 12 of *Quantum Reality* by Nick Herbert for complete analyses of this historically important experiment.

A two-stage atomic cascade emits entangled photons (A and B) in opposite directions with the same circular polarization according to the observers in their path.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[|L\rangle_A |L\rangle_B + |R\rangle_A |R\rangle_B\right]$$

The experiment involves the measurement of photon polarization states in the vertical/horizontal measurement basis, and allows that the polarization analyzers (PAs) can be oriented at different angles ***a*** and ***b***. (The figure below is taken from Chapter 4 of Baggott's book.)



Naturally the bipartate photon wave function is identical in both the circular or linear polarization bases.

Left circular polarization: $L = \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 \\ i \end{pmatrix}$ Right circular polarization: $R = \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 \\ -i \end{pmatrix}$

Vertical polarization: $V = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ Horizontal polarization: $H = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[|L\rangle_A |L\rangle_B + |R\rangle_A |R\rangle_B\right] = \frac{1}{2\sqrt{2}}\left[\begin{pmatrix} 1 \\ i \end{pmatrix}_A \otimes \begin{pmatrix} 1 \\ i \end{pmatrix}_B + \begin{pmatrix} 1 \\ -i \end{pmatrix}_A \otimes \begin{pmatrix} 1 \\ -i \end{pmatrix}_B\right] = \frac{1}{2\sqrt{2}}\left[\begin{pmatrix} 1 \\ i \\ i \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ -i \\ -i \\ -1 \end{pmatrix}\right] = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[|V\rangle_A |V\rangle_B - |H\rangle_A |H\rangle_B\right] = \frac{1}{2\sqrt{2}}\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}_A \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}_B - \begin{pmatrix} 0 \\ 1 \end{pmatrix}_A \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}_B\right] = \frac{1}{2\sqrt{2}}\left[\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}\right] = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

There are four measurement outcomes: both photons are vertically polarized, both are horizontally polarized, one is vertical and the other horizontal, and vice versa. The tensor representation of these measurement states are provided below.

$$|VV\rangle = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad |VH\rangle = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \quad |HV\rangle = \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \quad |HH\rangle = \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

We now write all states, Ψ and the measurement states, in Mathcad's vector format.

$$\Psi := \frac{1}{\sqrt{2}} \cdot \begin{pmatrix}1\\0\\0\\-1\end{pmatrix} \qquad VV := \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \qquad VH := \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \qquad HV := \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \qquad HH := \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

Next, the operator representing the rotation of PA_1 by angle ***a*** clockwise and PA_2 by angle ***b*** counter-clockwise (so that the PAs turn in the same direction) is constructed using matrix tensor multiplication. **Kronecker** is Mathcad's command for tensor matrix multiplication.

$$\mathrm{RotOp}(a,b) := \mathrm{kronecker}\left[\begin{pmatrix}\cos(a) & \sin(a)\\ -\sin(a) & \cos(a)\end{pmatrix}, \begin{pmatrix}\cos(b) & -\sin(b)\\ \sin(b) & \cos(b)\end{pmatrix}\right]$$

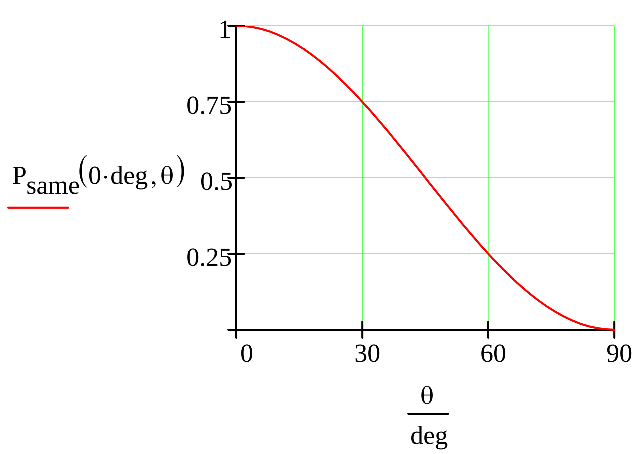
The probability that the detectors will behave the same or differently is calculated as follows.

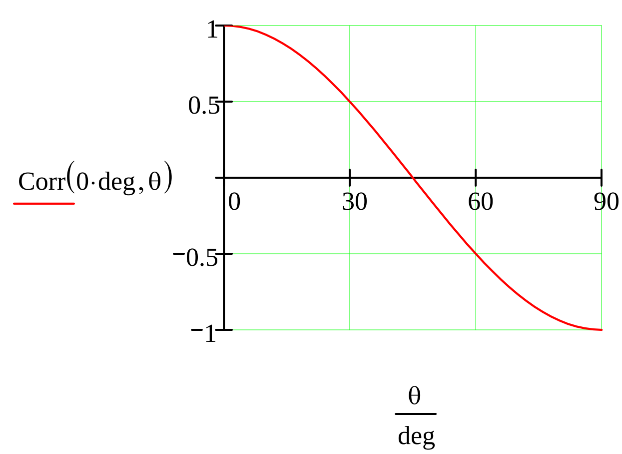
$$P_{same}(a,b) := \left(VV^T \cdot \mathrm{RotOp}(a,b) \cdot \Psi\right)^2 + \left(HH^T \cdot \mathrm{RotOp}(a,b) \cdot \Psi\right)^2$$

$$P_{diff}(a,b) := \left(VH^T \cdot \mathrm{RotOp}(a,b) \cdot \Psi\right)^2 + \left(HV^T \cdot \mathrm{RotOp}(a,b) \cdot \Psi\right)^2$$

The expectation value as a function of the relative orientation of the polarization detectors is the difference between these two expressions. This is generally called the correlation function. In other words, the composite eigenvalues are: ++ = -- = +1 (same) and +- = -+ = -1 (diff).

$$\mathrm{Corr}(a,b) := P_{same}(a,b) - P_{diff}(a,b) \qquad \theta := 0\cdot deg, 2\cdot deg\,..\,90\cdot deg$$





These graphical representations of the Aspect experiment are in agreement with those presented in Aspect's paper and also in *The Quantum Challenge*, *The Meaning of Quantum Theory*, and *Quantum Reality*.

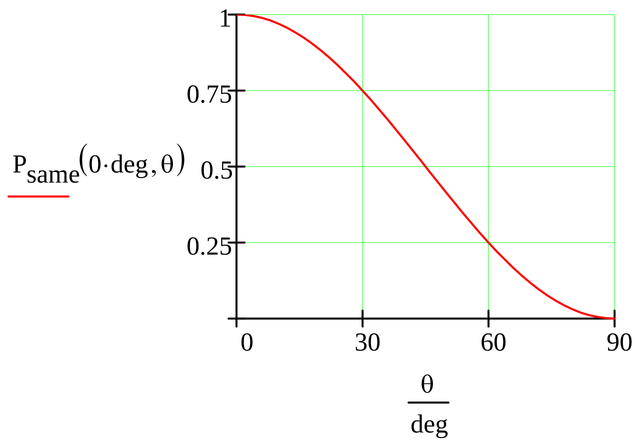
These calculations are now repeated for the three other Bell states.

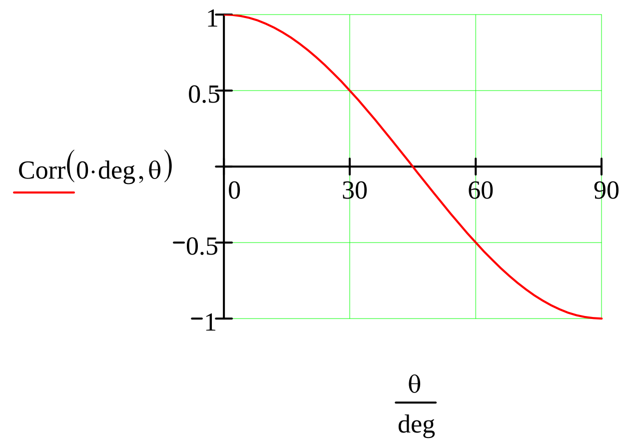
$$\Psi := \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 & 0 & 0 & 1 \end{pmatrix}^T$$

$$P_{same}(a, b) := \left(VV^T \cdot RotOp(a, b) \cdot \Psi\right)^2 + \left(HH^T \cdot RotOp(a, b) \cdot \Psi\right)^2$$

$$P_{diff}(a, b) := \left(VH^T \cdot RotOp(a, b) \cdot \Psi\right)^2 + \left(HV^T \cdot RotOp(a, b) \cdot \Psi\right)^2$$

$$Corr(a, b) := P_{same}(a, b) - P_{diff}(a, b)$$



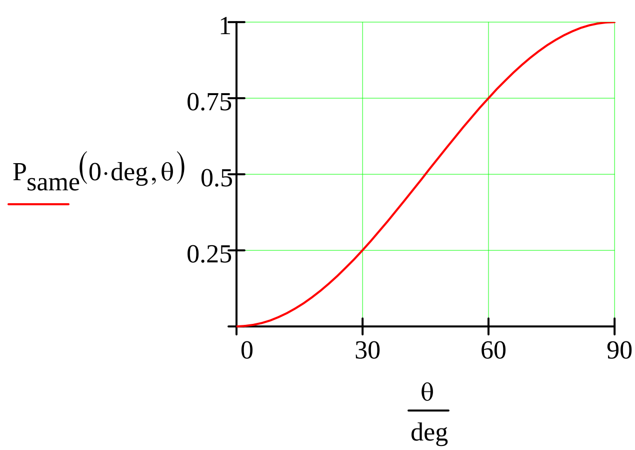


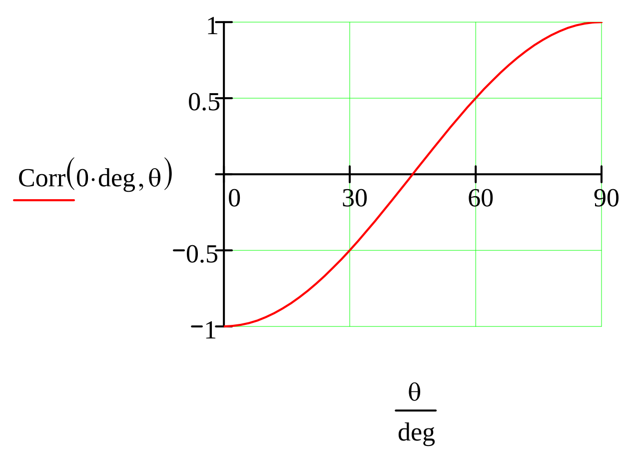
$$\Psi := \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 0 & 1 & 1 & 0 \end{pmatrix}^T$$

$$P_{same}(a, b) := \left(VV^T \cdot RotOp(a, b) \cdot \Psi\right)^2 + \left(HH^T \cdot RotOp(a, b) \cdot \Psi\right)^2$$

$$P_{diff}(a, b) := \left(VH^T \cdot RotOp(a, b) \cdot \Psi\right)^2 + \left(HV^T \cdot RotOp(a, b) \cdot \Psi\right)^2$$

$$Corr(a, b) := P_{same}(a, b) - P_{diff}(a, b)$$





$$\Psi := \frac{1}{\sqrt{2}} \cdot (0 \quad 1 \quad -1 \quad 0)^T$$

$$P_{same}(a,b) := \left(VV^T \cdot RotOp(a,b) \cdot \Psi\right)^2 + \left(HH^T \cdot RotOp(a,b) \cdot \Psi\right)^2$$

$$P_{diff}(a,b) := \left(VH^T \cdot RotOp(a,b) \cdot \Psi\right)^2 + \left(HV^T \cdot RotOp(a,b) \cdot \Psi\right)^2$$

$$Corr(a,b) := P_{same}(a,b) - P_{diff}(a,b)$$