

Numerical Solutions for the Finite Potential Well

Schrodinger's equation is integrated numerically for the first three energy states for a finite potential well. The integration algorithm is taken from J. C. Hansen, *J. Chem. Educ. Software*, **8C2**, 1996.

Set parameters:

$$\begin{array}{llll}
 n := 100 & x_{\min} := -3 & x_{\max} := 3 & \Delta := \frac{x_{\max} - x_{\min}}{n - 1} \\
 \mu := 1 & lb := -1 & rb := 1 & V_0 := 4
 \end{array}$$

Calculate position vector, the potential energy matrix, and the kinetic energy matrix. Then combine them into a total energy matrix.

$$\begin{array}{lll}
 i := 1..n & j := 1..n & x_i := x_{\min} + (i - 1) \cdot \Delta \\
 V_{i,i} := \text{if}[(x_i \geq lb) \cdot (x_i \leq rb), 0, V_0] & T_{i,j} := \text{if} \left[i = j, \frac{\pi^2}{6 \cdot \mu \cdot \Delta^2}, \frac{(-1)^{i-j}}{(i-j)^2 \cdot \mu \cdot \Delta^2} \right] &
 \end{array}$$

Form Hamiltonian energy matrix: $H := T + V$

Find eigenvalues: $E := \text{sort}(\text{eigenvals}(H))$ Display three eigenvalues: $m := 1..3$

$E_m =$

0.63423174
2.39691438
4.4105828

Calculate associated eigenfunctions: $k := 1..2$ $\Psi(k) := \text{eigenvec}(H, E_k)$

Plot the potential energy and bound state eigenfunctions: $V_{\text{pot}_i} := V_{i,i}$

