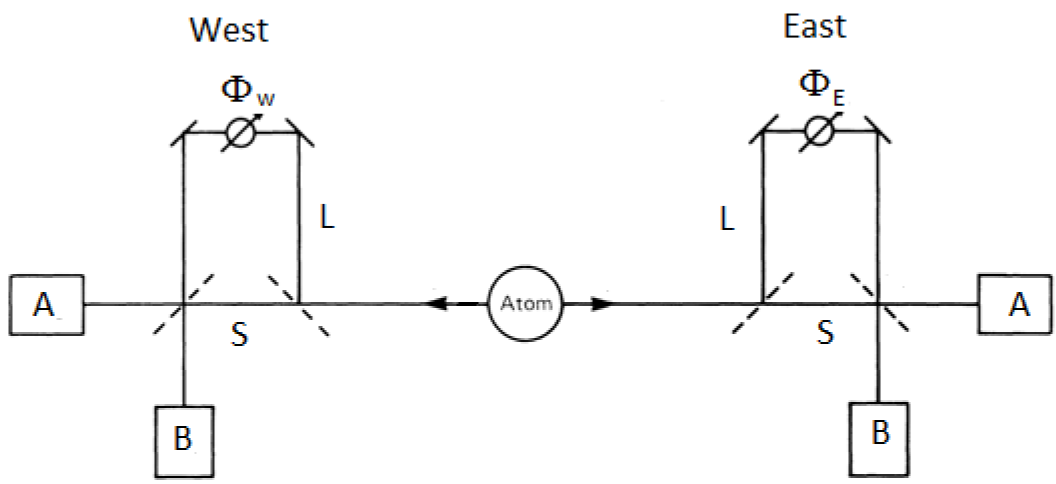
Analysis of a Two-photon Interferometer

Frank Rioux

In the two-photon interferometer shown below, an atom simultaneously emits identical photons to the east and west. Two paths (long and short) are available to detectors A and B in the east and west arms of the interferometer. The short paths are the same on both sides, but the long paths can have different lengths causing phase differences for the east and west directions.



Labeling the photons 1 and 2, and the directions W and E, the entangled photon wave function is:

$$\frac{1}{\sqrt{2}}\cdot\left(W_1\cdot E_2 + E_1\cdot W_2\right)$$

At the beam splitters (dashed diagonal lines), the probability amplitude for transmission is $\frac{1}{\sqrt{2}}$ and the probability amplitude for reflection is $\frac{i}{\sqrt{2}}$. By convention a $\pi/2$ phase shift is assigned to reflection. Armed with this information it in possible to trace the evolution of the initial entangled state.

After the first beam splitter in each arm we have:

$$W_1 = \frac{1}{\sqrt{2}}\cdot\left(S_{w1} + i\cdot\exp\left(i\phi_w\right)\cdot L_{w1}\right) \qquad E_1 = \frac{1}{\sqrt{2}}\cdot\left(S_{e1} + i\cdot\exp\left(i\phi_e\right)\cdot L_{e1}\right)$$

$$W_2 = \frac{1}{\sqrt{2}}\cdot\left(S_{w2} + i\cdot\exp\left(i\phi_w\right)\cdot L_{w2}\right) \qquad E_2 = \frac{1}{\sqrt{2}}\cdot\left(S_{e2} + i\cdot\exp\left(i\phi_e\right)\cdot L_{e2}\right)$$

The second beam splitter has the following effect on the eight terms above:

$$S_{w1} = \frac{1}{\sqrt{2}}\cdot\left(A_{w1} + i\cdot B_{w1}\right) \quad L_{w1} = \frac{1}{\sqrt{2}}\cdot\left(i\cdot A_{w1} + B_{w1}\right) \quad S_{w2} = \frac{1}{\sqrt{2}}\cdot\left(A_{w2} + i\cdot B_{w2}\right) \quad L_{w2} = \frac{1}{\sqrt{2}}\cdot\left(i\cdot A_{w2} + B_{w2}\right)$$

$$S_{e1} = \frac{1}{\sqrt{2}}\cdot\left(A_{e1} + i\cdot B_{e1}\right) \quad L_{e1} = \frac{1}{\sqrt{2}}\cdot\left(i\cdot A_{e1} + B_{e1}\right) \quad S_{e2} = \frac{1}{\sqrt{2}}\cdot\left(A_{e2} + i\cdot B_{e2}\right) \quad L_{e2} = \frac{1}{\sqrt{2}}\cdot\left(i\cdot A_{e2} + B_{e2}\right)$$

Mathcad facilitates the substitution of these 12 expressions into the original entangled two-photon wave function as is shown below. To expedite interpretation we choose specific values for the relative phases of the photons in the branches of the interferometer. If the L paths both have $\phi = 0$, they have the same length and the same length as the S paths. The photons always arrive at the B detectors. Other simple cases are summarized in the following table. However, most L-path phase relationships lead to complicated output wave functions.

$$\begin{bmatrix} \phi_w & \phi_e & \text{WF} \\ 0 & 0 & \frac{-1}{2}\cdot 2^{\frac{1}{2}}\cdot\left(B_{w1}\cdot B_{e2} + B_{e1}\cdot B_{w2}\right) \\ 0 & \pi & \frac{1}{2}\cdot i\cdot 2^{\frac{1}{2}}\cdot\left(B_{w1}\cdot A_{e2} + A_{e1}\cdot B_{w2}\right) \\ \pi & \pi & \frac{1}{2}\cdot 2^{\frac{1}{2}}\cdot\left(A_{w1}\cdot A_{e2} + A_{e1}\cdot A_{w2}\right) \end{bmatrix}$$

$$\phi_w := 0 \qquad \phi_e := 0$$

$$\frac{1}{\sqrt{2}}\cdot\left(W_1\cdot E_2 + E_1\cdot W_2\right) \left| \begin{aligned} &\text{substitute}, W_1 = \frac{1}{\sqrt{2}}\cdot\left(S_{w1} + i\cdot\exp\left(i\phi_w\right)\cdot L_{w1}\right) \\ &\text{substitute}, W_2 = \frac{1}{\sqrt{2}}\cdot\left(S_{w2} + i\cdot\exp\left(i\phi_w\right)\cdot L_{w2}\right) \\ &\text{substitute}, E_1 = \frac{1}{\sqrt{2}}\cdot\left(S_{e1} + i\cdot\exp\left(i\phi_e\right)\cdot L_{e1}\right) \\ &\text{substitute}, E_2 = \frac{1}{\sqrt{2}}\cdot\left(S_{e2} + i\cdot\exp\left(i\phi_e\right)\cdot L_{e2}\right) \\ &\text{substitute}, S_{w1} = \frac{1}{\sqrt{2}}\cdot\left(A_{w1} + i\cdot B_{w1}\right) \\ &\text{substitute}, L_{w1} = \frac{1}{\sqrt{2}}\cdot\left(i\cdot A_{w1} + B_{w1}\right) \\ &\text{substitute}, S_{w2} = \frac{1}{\sqrt{2}}\cdot\left(A_{w2} + i\cdot B_{w2}\right) \\ &\text{substitute}, L_{w2} = \frac{1}{\sqrt{2}}\cdot\left(i\cdot A_{w2} + B_{w2}\right) \\ &\text{substitute}, S_{e1} = \frac{1}{\sqrt{2}}\cdot\left(A_{e1} + i\cdot B_{e1}\right) \\ &\text{substitute}, L_{e1} = \frac{1}{\sqrt{2}}\cdot\left(i\cdot A_{e1} + B_{e1}\right) \\ &\text{substitute}, S_{e2} = \frac{1}{\sqrt{2}}\cdot\left(A_{e2} + i\cdot B_{e2}\right) \\ &\text{substitute}, L_{e2} = \frac{1}{\sqrt{2}}\cdot\left(i\cdot A_{e2} + B_{e2}\right) \\ &\text{simplify} \end{aligned} \right. \rightarrow \frac{-1}{2}\cdot 2^{\frac{1}{2}}\cdot\left(B_{w1}\cdot B_{e2} + B_{e1}\cdot B_{w2}\right)$$