Two-electron Interference

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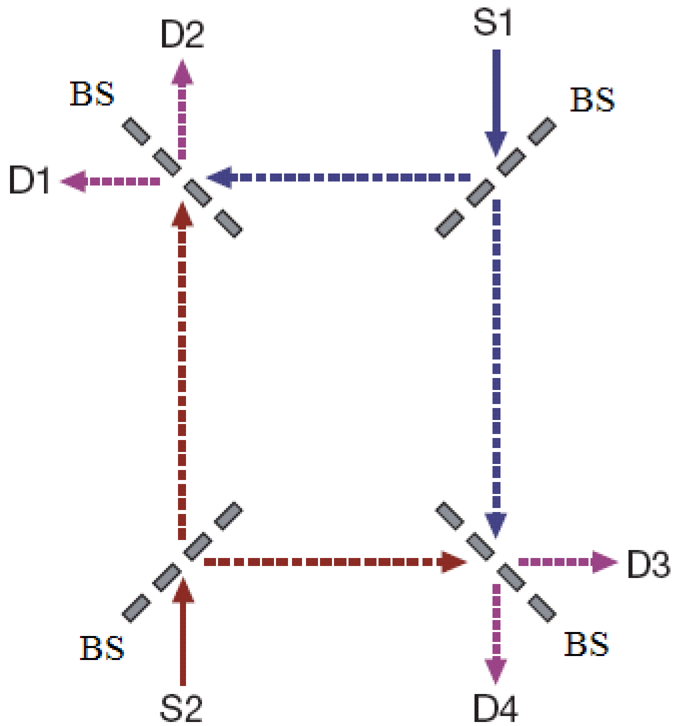
This tutorial provides a simplified analysis of a recent experiment describing the "interference between two indistinguishable electrons from independent sources."(*1*) A schematic of the interferometer used in this experiment is shown below.



S1 and S2 are the electron sources, BS labels the four 50-50 beam splitters and D1, D2, etc. are the electron detectors.

To simplify the analysis it will be assumed that the arms of the interferometer are of equal length. This will make it unnecessary to consider the accumulation of phase differences due to unequal path distances to the detectors.

The sources simultaneously inject electrons into the interferometer. The presence of the beam splitters provides each source beam access to all four detectors. For example, the source wave functions written in the basis of the detectors are,

$$|S1\rangle \rightarrow \frac{1}{2}\left[i|D1\rangle - |D2\rangle + i|D3\rangle + |D4\rangle\right]$$

$$|S2\rangle \rightarrow \frac{1}{2}\left[i|D1\rangle + |D2\rangle + i|D3\rangle - |D4\rangle\right]$$

Following the usual convention a π/2 phase shift (represented by i) is assigned to reflection at the beam splitter. It should be noted that there is no opportunity for electron self-interference because the individual electron beams are not recombined. This means the source of interference in this experiment is due to the indistinguishability of the electrons and the fermionic character of the total wave function which is an entangled two-electron state.

$$|\Psi\rangle_{tot} = \frac{1}{\sqrt{2}}\left[|S1\rangle_a |S2\rangle_b - |S2\rangle_a |S1\rangle_b\right]$$

Substitution of the first two equations into the third yields,

$$|\Psi\rangle_{tot} = \frac{i\sqrt{2}}{4}\left[|D1\rangle_a|D2\rangle_b - |D2\rangle_a|D1\rangle_b - |D2\rangle_a|D3\rangle_b - |D1\rangle_a|D4\rangle_b - |D3\rangle_a|D4\rangle_b + |D4\rangle_a|D1\rangle_b + |D3\rangle_a|D2\rangle_b + |D4\rangle_a|D3\rangle_b\right]$$

where the subscripts ***a*** and ***b*** label the electrons.

This result predicts that the electrons never arrive at the same detector, and that the D1-D3 and D2-D4 coincidences are not observed. This is in agreement with the more general results reported by Neder, et al. (*1*) See, for example, their equation (3) for $\phi1 = \phi2 = \phi3 = \phi4 = 0$.

Neder, et al. comment as follows on the distinction between single- and two-particle interference.

> Very much like the ubiquitous quantum interference of a single particle with itself, quantum interference of two independent, but indistinquishable, particles is also possible. For a single particle, the interference is between the amplitudes of the particle's wave function, whereas the interference between two particles is a direct result of quantum exchange statistics. Such interference is observed only in the joint probability of finding the particles in two separated detectors, after they were injected from spatially separated and independent sources.

While it is customary to talk about particle interference, as these authors do, Nobel Laureate Roy Glauber recommended more careful language. (2)

> The things that interfere in quantum mechanics are not particles. They are probability amplitudes for certain events. It is the fact that probability amplitudes add up like complex numbers that is responsible for all quantum mechanical interferences.

The insightfulness of this observation is clearly revealed in the mathematical underpinnings of this experiment.

Literature cited:

1. Neder, I. *et al. Nature* **448**, 333-337 (2007)
2. Glauber, J. *AmericanJournal of Physics*, **63(1)**, 12 (1995).