Is a Two-dimensional Fibonacci Array a Quasilattice?

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A two-dimensional Fibonacci lattice lacks translational periodicity but has a discrete diffraction pattern, just like a quasicrystal. However, it does not fit the definition of a quasilattice because it does not posses one of the 'forbidden' n-fold rotational symmetries (n = 5 or greater than 6) that are characteristic of quasicrystals and incompatible with translational periodicity. R. Lifshitz[1], therefore, recommends that the symmetry requirement be relaxed so that two- and three-dimensional Fibonacci lattices can have quasilattice stature.

A one-dimensional Fibonacci grid consists of a sequence of long (L) and short (S) segments such as LSLLSLSLLS.... with L/S = 1.618, the golden ratio. A two-dimensional array is created by superimposing two such grids at a 90^o angle and placing atomic scatterers at the vertices.

Dimension of grid: $A := 10$ $\quad m := 1 .. A$ $\quad n := 1 .. A$ $\quad \tau := \frac{1+\sqrt{5}}{2}$

Calculate the coordinates of the Fibonacci vertices in a two-dimensional lattice (see Lifshitz).

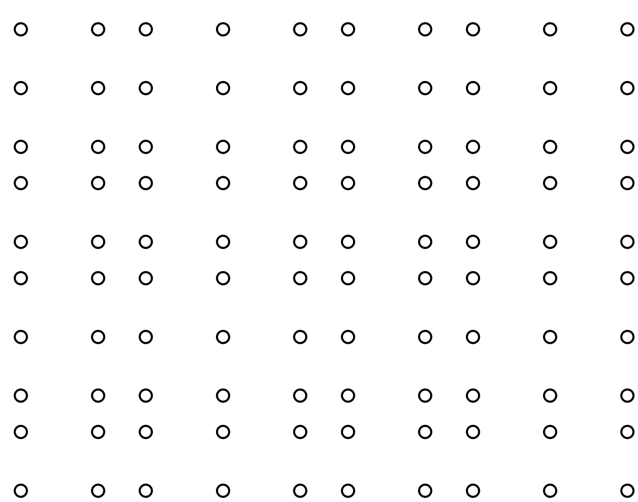
$$x_m := \mathrm{floor}\left(\frac{m}{\tau}\right)\cdot\tau + \left(m - \mathrm{floor}\left(\frac{m}{\tau}\right)\right) - 1 \qquad y_n := \mathrm{floor}\left(\frac{n}{\tau}\right)\cdot\tau + \left(n - \mathrm{floor}\left(\frac{n}{\tau}\right)\right) - 1$$

$$x^T = (0 \quad 1.618 \quad 2.618 \quad 4.236 \quad 5.854 \quad 6.854 \quad 8.472 \quad 9.472 \quad 11.09 \quad 12.708)$$

$$y^T = (0 \quad 1.618 \quad 2.618 \quad 4.236 \quad 5.854 \quad 6.854 \quad 8.472 \quad 9.472 \quad 11.09 \quad 12.708)$$

Display the two-dimensional Fibonacci array:

2D Fibonacci Lattice



The diffraction pattern is the Fourier transform of the spatial Fibonacci array into the momentum representation.

Calculate momentum-space wave function:

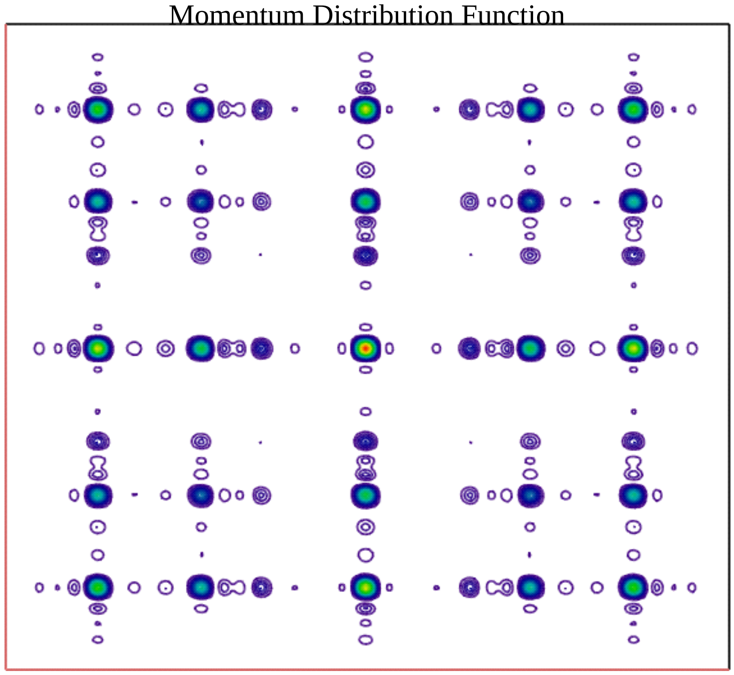
$$\Phi(p_x, p_y) := \frac{1}{2\cdot\pi}\cdot\sum_{m=1}^{A} \exp(-i\cdot p_x\cdot x_m)\cdot\sum_{n=1}^{A} \exp(-i\cdot p_y\cdot y_n)$$

Display momentum-space distribution function (diffraction pattern):

$\Delta := 10$ $\quad N := 200$ $\quad j := 1..N$ $\quad p_{x_j} := -\Delta + \frac{2\cdot\Delta\cdot j}{N}$ $\quad k := 1..N$ $\quad p_{y_k} := -\Delta + \frac{2\cdot\Delta\cdot k}{N}$

$$\text{DiffractionPattern}_{j,k} := \left(\left|\Phi\left(p_{x_j}, p_{y_k}\right)\right|\right)^2$$

Momentum Distribution Function



DiffractionPattern

1. R. Lifshitz, "The square Fibonacci tiling," *Journal of Alloys and Compounds* **342**, 186-190 (2002).