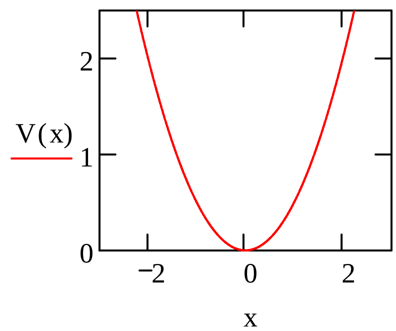
Variation Method for a Particle Traped in Harmonic Potential Well

Define potential energy: $V(x) := \frac{x^2}{2}$

Display potential energy:



Choose trial wave function: $\Psi(x,\beta) := \sqrt{\frac{\beta}{2}} \cdot \mathrm{sech}(\beta \cdot x)$

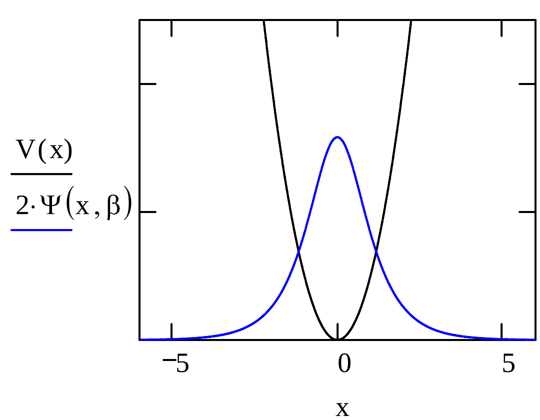
Set up variational energy integral:

$$E(\beta) := \int_{-\infty}^{\infty} \Psi(x,\beta) \cdot \frac{-1}{2} \cdot \frac{d^2}{dx^2}\Psi(x,\beta)\,dx + \int_{-\infty}^{\infty} V(x) \cdot \Psi(x,\beta)^2\,dx \;\Bigg|\; \begin{array}{l}\text{assume}, \beta > 0 \\ \text{simplify}\end{array} \rightarrow \frac{1}{24} \cdot \frac{4 \cdot \beta^4 + \pi^2}{\beta^2}$$

Minimize the energy integral with respect to the variational parameter, β.

$\beta := .2$ $\quad \beta := \mathrm{Minimize}(E,\beta)$ $\quad \beta = 1.253$ $\quad E(\beta) = 0.524$

Display wave function in the potential well.



Calculate the probability that the particle is in the potential barrier.

$$2\cdot\int_0^{\infty}\Psi(x,\beta)^2\,dx = 1$$

Define quantum mechanical tunneling.

Tunneling occurs when a quon (a quantum mchanical particle) has probability of being in a nonclassical region. In other words, a region in which the total energy is less than the potential energy.

Calculate the probability that tunneling is occuring.

Calculate the classical turning point.

$$\frac{x^2}{2} = 0.524 \;\Big|\; \begin{matrix}\text{solve}, x \\ \text{float}, 4\end{matrix} \rightarrow \begin{pmatrix}-1.024 \\ 1.024\end{pmatrix}$$

$$2\cdot\int_{1.024}^{\infty}\Psi(x,\beta)^2\,dx = 0.143$$

Calculate the kinetic and potential enegy contriutions to the total energy.

Kinetic energy:

$$\int_{-\infty}^{\infty}\Psi(x,\beta)\cdot\frac{-1}{2}\cdot\frac{d^2}{dx^2}\Psi(x,\beta)\,dx = 0.262$$

Potential energy:

$$\int_{-\infty}^{\infty}V(x)\cdot\Psi(x,\beta)^2\,dx = 0.262$$

Is the virial theorem satisfied?

Yes, for the harmonic potential the virial theorem is T = V = E/2.