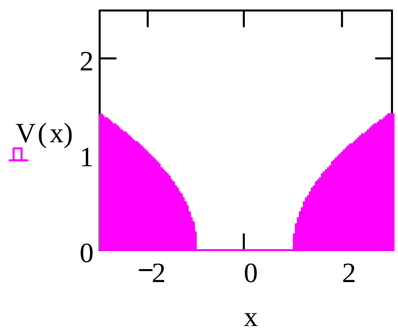
Approximate Quantum Mechanical Methods
Variation Method
Particle in a 1D Symmetric Potential Well

Define potential energy: $V(x) := \text{if}\left[(x \geq -1)\cdot(x \leq 1), 0, \sqrt{|x| - 1}\right]$

Display potential energy:



Choose trial wave function: $\Psi(x,\beta) := \left(\frac{2\cdot\beta}{\pi}\right)^{\frac{1}{4}}\cdot\exp\left(-\beta\cdot x^2\right)$

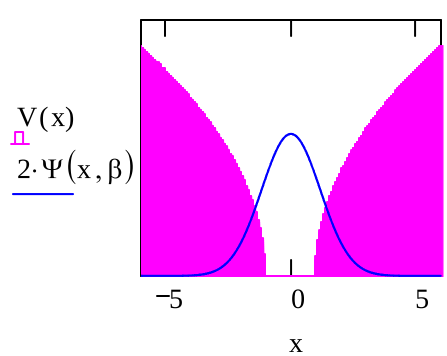
Evaluate the variational integral:

$$E(\beta) := \int_{-\infty}^{\infty} \Psi(x,\beta)\cdot -\frac{1}{2}\cdot\frac{d^2}{dx^2}\Psi(x,\beta)\,dx + \int_{-\infty}^{\infty} V(x)\cdot\Psi(x,\beta)^2\,dx$$

Minimize the energy integral with respect to the variational parameter, β.

$\beta := .2$ $\beta := \text{Minimize}(E,\beta)$ $\beta = 0.363$ $E(\beta) = 0.313$

Display wave function in the potential well.



Calculate the probability that the particle is in the potential barrier.

$$2 \cdot \int_{1}^{\infty} \Psi(x,\beta)^2 \, dx = 0.228$$

Define quantum mechanical tunneling.

Tunneling occurs when a quon (a quantum mchanical particle) has probability of being in a nonclassical region. In other words, a region in which the total energy is less than the potential energy.

Calculate the probability that tunneling is occuring.

$$|x| - 1 = 0.313^2 \left| \begin{matrix} \text{solve}, x \\ \text{float}, 4 \end{matrix} \right. \rightarrow \begin{pmatrix} 1.098 \\ -1.098 \end{pmatrix}$$

$$2 \cdot \int_{1.098}^{\infty} \Psi(x,\beta)^2 \, dx = 0.186$$

Calculate the kinetic and potential enegy contriutions to the total energy.

Kinetic energy:

$$\int_{-\infty}^{\infty} \Psi(x,\beta) \cdot -\frac{1}{2} \cdot \frac{d^2}{dx^2} \Psi(x,\beta) \, dx = 0.182$$

Potential energy:

$$\int_{-\infty}^{\infty} V(x) \cdot \Psi(x,\beta)^2 \, dx = 0.131$$