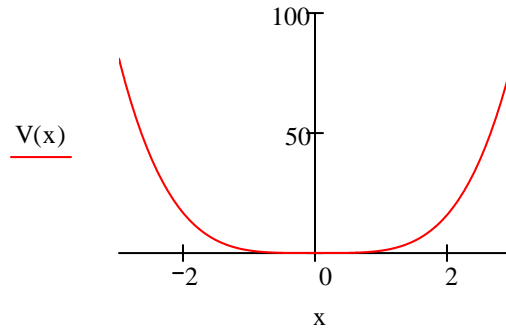


Variation Method Using the Wigner Function: The Quartic Oscillator

Frank Rioux

Define potential energy: $V(x) := x^4$

Display potential energy:



Choose trial wave function: $\Psi(x, \beta) := \left(\frac{2 \cdot \beta}{\pi}\right)^{\frac{1}{4}} \cdot \exp(-\beta \cdot x^2)$

Calculate the Wigner distribution function:

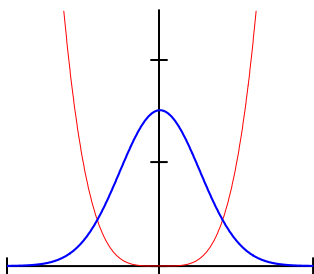
$$W(x, p, \beta) := \frac{1}{2\pi} \cdot \int_{-\infty}^{\infty} \Psi\left(x + \frac{s}{2}, \beta\right) \cdot \exp(i \cdot s \cdot p) \cdot \Psi\left(x - \frac{s}{2}, \beta\right) ds \quad \left| \begin{array}{l} \text{simplify} \\ \text{assume, } \beta > 0 \end{array} \right. \rightarrow \frac{1}{\pi} \cdot e^{\frac{-1}{2} \cdot \frac{4 \cdot \beta^2 \cdot x^2 + p^2}{\beta}}$$

Evaluate the variational integral: $E(\beta) := \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(x, p, \beta) \cdot \left(\frac{p^2}{2} + V(x)\right) dx dp$

Minimize the energy integral with respect to the variational parameter, β .

$$\beta := 1 \quad \beta := \text{Minimize}(E, \beta) \quad \beta = 0.90856 \quad E(\beta) = 0.68142$$

Calculate and display the coordinate distribution function: $P_X(x, \beta) := \int_{-\infty}^{\infty} W(x, p, \beta) dp$

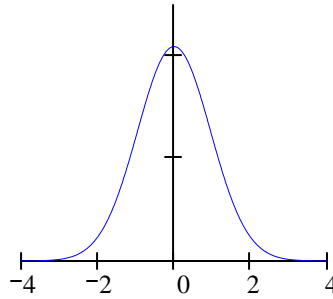


Classical turning point: $x_{cl} := 0.681^{\frac{1}{4}} \quad x_{cl} = 0.90842$

Probability that tunneling is occurring: $2 \cdot \int_{0.908}^{\infty} P_X(x, \beta) dx = 0.08345$

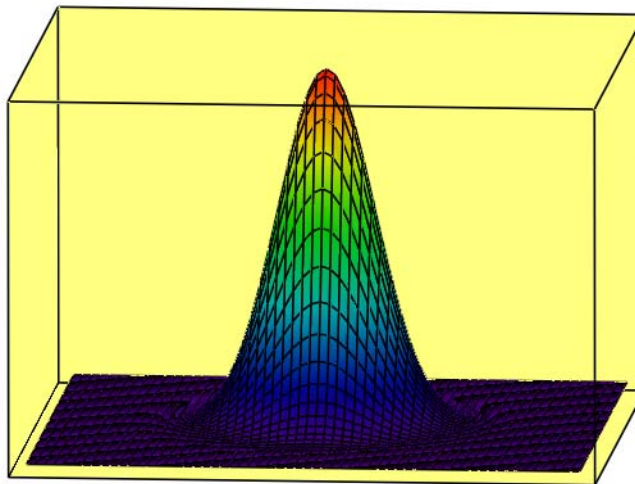
Calculate and display the momentum distribution function:

$$P(p, \beta) := \int_{-\infty}^{\infty} W(x, p, \beta) dx$$



Display the Wigner distribution function:

$$N := 60 \quad i := 0..N \quad x_i := -3 + \frac{6 \cdot i}{N} \quad j := 0..N \quad p_j := -5 + \frac{10 \cdot j}{N} \quad \text{Wigner}_{i,j} := W(x_i, p_j, \beta)$$



Wigner