Variation Method Using the Wigner Function: The Harmonic Oscillator

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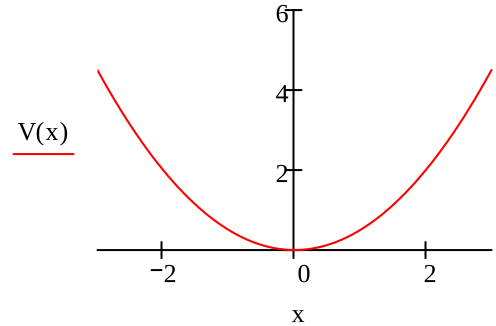
Define potential energy: $V(x) := \frac{x^2}{2}$

Display potential energy:



Choose trial wave function: $\Psi(x,\beta) := \left(\frac{2\cdot\beta}{\pi}\right)^{\frac{1}{4}}\cdot\exp\left(-\beta\cdot x^2\right)$

Calculate the Wigner distribution function:

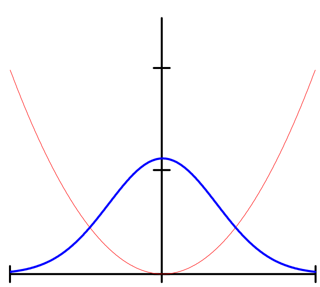
$$W(x,p,\beta) := \frac{1}{2\pi}\cdot\int_{-\infty}^{\infty}\Psi\left(x+\frac{s}{2},\beta\right)\cdot\exp(i\cdot s\cdot p)\cdot\Psi\left(x-\frac{s}{2},\beta\right)\,ds \;\Bigg|\begin{array}{l}\text{simplify}\\ \text{assume}, \beta > 0\end{array} \rightarrow \frac{1}{\pi}\cdot e^{\frac{-1}{2}\cdot\frac{4\cdot\beta^2\cdot x^2+p^2}{\beta}}$$

Evaluate the variational integral: $E(\beta) := \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W(x,p,\beta)\cdot\left(\frac{p^2}{2}+V(x)\right)\,dx\,dp$

Minimize the energy integral with respect to the variational parameter, β.

$\beta := 1$ $\qquad \beta := \text{Minimize}(E,\beta)$ $\qquad \beta = 0.5$ $\qquad E(\beta) = 0.5$

Calculate and display the coordinate distribution function: $Px(x,\beta) := \int_{-\infty}^{\infty} W(x,p,\beta)\,dp$

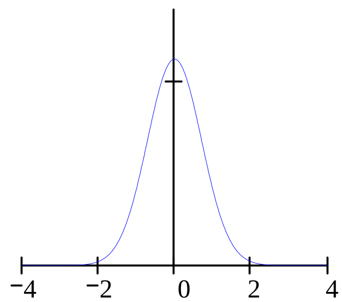


Classical turning point: $x_{cl} := 0.5^{\frac{1}{2}}$ $\qquad x_{cl} = 0.707$

Probability that tunneling is occuring: $2\cdot\int_{0.707}^{\infty} Px(x,\beta)\,dx = 0.317$

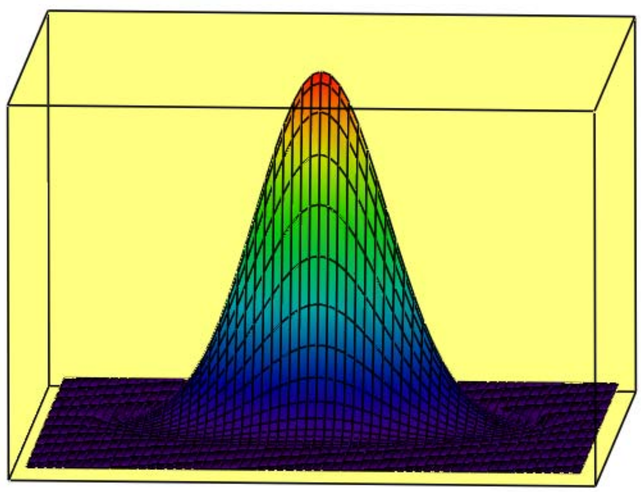
Calculate and display the momentum distribution function:

$$Pp(p,\beta) := \int_{-\infty}^{\infty} W(x,p,\beta)\,dx$$



Display the Wigner distribution function:

$N := 60$ $i := 0..N$ $x_i := -3 + \frac{6\cdot i}{N}$ $j := 0..N$ $p_j := -5 + \frac{10\cdot j}{N}$ $Wigner_{i,j} := W(x_i, p_j, \beta)$



Wigner