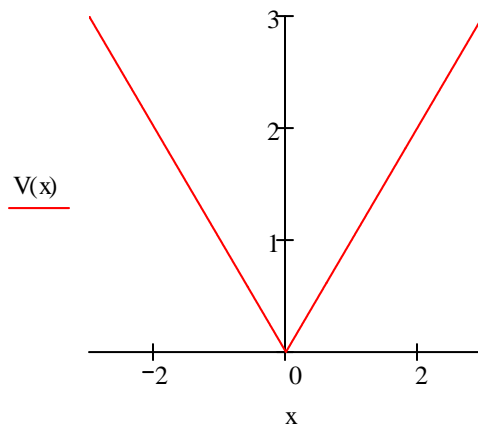


# Variation Method Using the Wigner Function: $V(x)=|x|$

Frank Rioux

Define potential energy:  $V(x) := |x|$

Display potential energy:



Choose trial wave function:  $\Psi(x, \beta) := \left(\frac{2 \cdot \beta}{\pi}\right)^{\frac{1}{4}} \cdot \exp(-\beta \cdot x^2)$

Calculate the Wigner distribution function:

$$W(x, p, \beta) := \frac{1}{2\pi} \cdot \int_{-\infty}^{\infty} \Psi\left(x + \frac{s}{2}, \beta\right) \cdot \exp(i \cdot s \cdot p) \cdot \Psi\left(x - \frac{s}{2}, \beta\right) ds \quad \left| \begin{array}{l} \text{simplify} \\ \text{assume, } \beta > 0 \end{array} \right. \rightarrow \frac{1}{\pi} \cdot e^{\frac{-1}{2} \cdot \frac{4 \cdot \beta^2 \cdot x^2 + p^2}{\beta}}$$

Evaluate the variational integral:

$$E(\beta) := \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(x, p, \beta) \cdot \left(\frac{p^2}{2} + V(x)\right) dx dp \quad \left| \begin{array}{l} \text{assume, } \beta > 0 \\ \text{simplify} \end{array} \right. \rightarrow \frac{1}{2} \cdot \frac{\frac{3}{2} \cdot \frac{1}{\pi} \cdot \frac{1}{\beta^2} + 2^2}{\frac{1}{\pi} \cdot \frac{1}{\beta^2}}$$

Minimize the energy integral with respect to the variational parameter,  $\beta$ .

$$\beta := 1$$

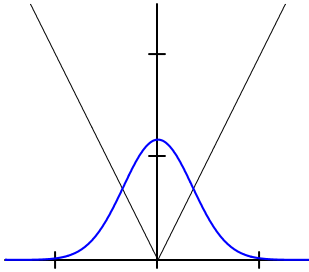
$$\beta := \text{Minimize}(E, \beta)$$

$$\beta = 0.542$$

$$E(\beta) = 0.813$$

Calculate and display the coordinate distribution function:

$$P_x(x, \beta) := \int_{-\infty}^{\infty} W(x, p, \beta) dp$$



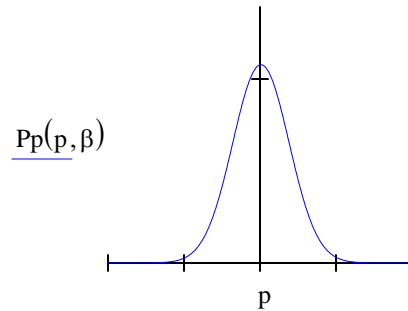
Classical turning points: +/- 0.813

Probability that tunneling is occurring:

$$2 \cdot \int_{0.813}^{\infty} P_x(x, \beta) dx = 0.231$$

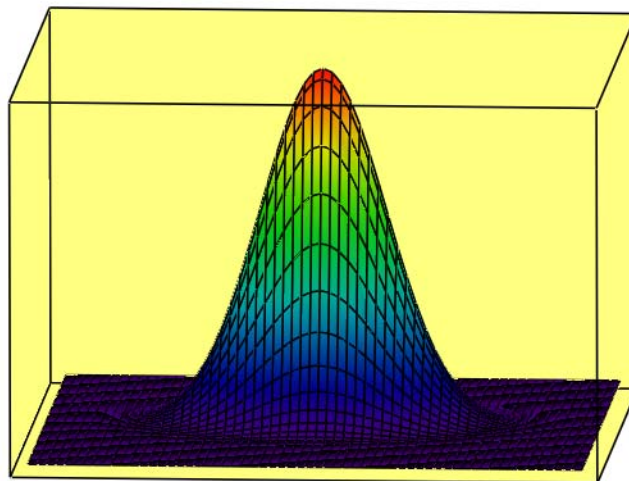
Calculate and display the momentum distribution function:

$$P_p(p, \beta) := \int_{-\infty}^{\infty} W(x, p, \beta) dx$$



Display the Wigner distribution function:

$$N := 60 \quad i := 0..N \quad x_i := -3 + \frac{6 \cdot i}{N} \quad j := 0..N \quad p_j := -5 + \frac{10 \cdot j}{N} \quad \text{Wigner}_{i,j} := W(x_i, p_j, \beta)$$



Wigner