Trial Wave Functions for Various Potentials

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This is list of functions and the potentials for which they would be suitable trial wave functions in a variation method calculation.

$$\Psi(x,\alpha) := 2\cdot\alpha^{\frac{3}{2}}\cdot x\cdot \exp(-\alpha\cdot x) \qquad \Psi(x,\alpha) := \left(\frac{128\cdot\alpha^3}{\pi}\right)^{\frac{1}{4}}\cdot x\cdot \exp\left(-\alpha\cdot x^2\right)$$

- Particle in a gravitational field V(z) = mgz (z = 0 to ∞)
- Particle confined by a linear potential V(x) = ax (x = 0 to ∞)
- One-dimensional atoms and ions V(x) = -Z/x (x = 0 to ∞)
- Particle in semi-infinite potential well V(x) = if[x ≤ a , 0 , b] (x = 0 to ∞)
- Particle in semi-harmonic potential well V(x) = kx^2 (x = 0 to ∞)

$$\Psi(x,\alpha) := \left(\frac{2\cdot\alpha}{\pi}\right)^{\frac{1}{4}}\cdot \exp\left(-\alpha\cdot x^2\right)$$

- Quartic oscillator V(x) = bx^4 (x = -∞ to ∞)
- Particle in the finite one-dimensional potential well $V(x) := \text{if}[(x \ge -1)\cdot(x \le 1), 0, 2]$ (x = -∞ to ∞)
- 1D Hydrogen atom ground state
- Harmonic oscillator ground state
- Particle in V(x) = | x | potential well

$$\Psi(x,\alpha) := \sqrt{\alpha}\cdot \exp(-\alpha\cdot |x|)$$

- This wavefunction is discontinuous at x = 0, so the following calculations must be made in momentum space
- Dirac hydrogen atom $V(x) := -\Delta(x)$
- Harmonic oscillator ground state
- Particle in V(x) = | x | potential well
- Quartic oscillator V(x) = bx^4 (x = -∞ to ∞)

$$\Psi(x) := \sqrt{30}\cdot x\cdot(1-x) \qquad \Gamma(x) := \sqrt{105}\cdot x\cdot(1-x)^2 \qquad \Theta(x) := \sqrt{105}\cdot x^2\cdot(1-x)$$

- Particle in a one-dimensional, one-bohr box
- Particle in a slanted one-dimensional box
- Particle in a semi-infinite potential well (change 1 to variational parameter)
- Particle in a gravitational field (change 1 to variational parameter)

$$\Phi(r,a) := (a-r) \qquad \Phi(r,a) := (a-r)^2 \qquad \Phi(r,a) := \frac{1}{\sqrt{2\cdot\pi\cdot a}}\cdot\frac{\sin\left(\frac{\pi\cdot r}{a}\right)}{r}$$

- Particle in a infinite spherical potential well of radius a
- Particle in a finite spherical potential well (treat **a** as a variational parameter)

$$\Psi(r, \beta) := \left(\frac{2 \cdot \beta}{\pi}\right)^{\frac{3}{4}} \cdot \exp\left(-\beta \cdot r^2\right)$$

- Particle in a finite spherical potential well
- Hydrogen atom ground state
- Helium atom ground state

$$\Psi(r, \beta) := \sqrt{\frac{3 \cdot \beta^3}{\pi^3}} \cdot \operatorname{sech}(\beta \cdot r)$$

- Particle in a finite potential well
- Hydrogen atom ground state
- Helium atom ground state

$$\Psi(x, \beta) := \sqrt{\frac{\beta}{2}} \cdot \operatorname{sech}(\beta \cdot x)$$

- Harmonic oscillator
- Quartic oscillator
- Particle in a gravitational field
- Particle in a finite potential well

$$\Psi(\alpha, x) := \frac{\sqrt{12 \cdot \alpha^3}}{\pi} \cdot x \cdot \operatorname{sech}(\alpha \cdot x)$$

- Particle in a semi-infinite potential well
- Particle in a gravitational field
- Particle in a linear potential well (same as above) $V(x) = ax$ ($x = 0$ to ∞)
- 1D hydrogen atom or one-electron ion

Some finite potential energy wells.

$$V(x) := \text{if}\left[(x \ge -1) \cdot (x \le 1), 0, V_0\right] \qquad V(x) := \text{if}\left[(x \ge -1) \cdot (x \le 1), 0, |x| - 1\right]$$

$$V(x) := \text{if}\left[(x \ge -1) \cdot (x \le 1), 0, \sqrt{|x| - 1}\right]$$

Some semi-infinite potential energy well.

$$V(x) := \text{if}(x \le a, 0, b) \qquad V(x) := \text{if}\left[(x \le 2), 0, \frac{5}{x}\right] \qquad V(x) := \text{if}\left[(x \le 2), 0, (x - 2)\right]$$

$$V(x) := \text{if}\left[(x \le 2), 0, \sqrt{x - 2}\right]$$