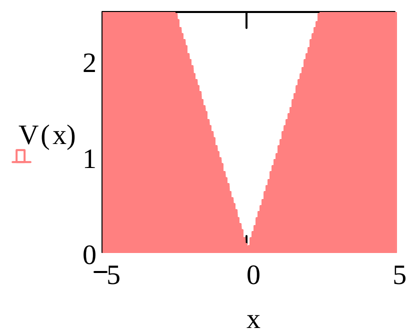
Approximate Quantum Mechanical Methods
Variation Method for a Particle in a 1D Ice Cream Cone

Define potential energy: $V(x) := |x|$

Display potential energy:



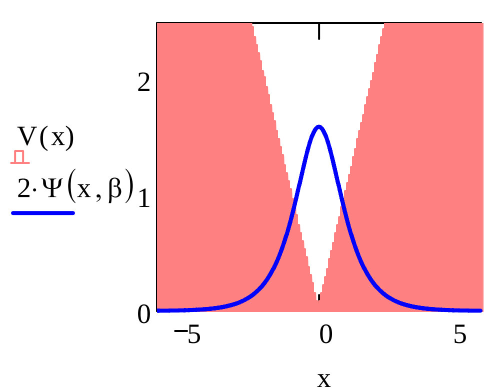
Choose trial wave function: $\Psi(x,\beta) := \sqrt{\frac{\beta}{2}} \cdot \text{sech}(\beta \cdot x)$

$$E(\beta) := \int_{-\infty}^{\infty} \Psi(x,\beta) \cdot \frac{-1}{2} \cdot \frac{d^2}{dx^2} \Psi(x,\beta)\, dx + \int_{-\infty}^{\infty} V(x) \cdot \Psi(x,\beta)^2\, dx$$

Minimize the energy integral with respect to the variational parameter, β.

$\beta := .2$ $\quad\beta := \text{Minimize}(E,\beta)$ $\quad\beta = 1.276$ $\quad E(\beta) = 0.815$

Display wave function in the potential well.



Calculate the probability that the particle is in the potential barrier.

$$2 \cdot \int_{0}^{\infty} \Psi(x,\beta)^2 \, dx = 1$$

Define quantum mechanical tunneling.

Tunneling occurs when a quon (a quantum mchanical particle) has probability of being in a nonclassical region. In other words, a region in which the total energy is less than the potential energy.

Calculate the probability that tunneling is occuring.

$$|x| = 0.815 \left| \begin{matrix} \text{solve}, x \\ \text{float}, 4 \end{matrix} \right. \rightarrow \begin{pmatrix} .8150 \\ -.8150 \end{pmatrix}$$

$$2 \cdot \int_{0.815}^{\infty} \Psi(x,\beta)^2 \, dx = 0.222$$

Calculate the kinetic and potential enegy contriutions to the total energy.

Kinetic energy:

$$\int_{-\infty}^{\infty} \Psi(x,\beta) \cdot \frac{-1}{2} \cdot \frac{d^2}{dx^2} \Psi(x,\beta) \, dx = 0.272$$

Potential energy:

$$\int_{-\infty}^{\infty} V(x) \cdot \Psi(x,\beta)^2 \, dx = 0.543$$