Hydrogen Atom Calculation Assuming the Electron is a Particle in a Sphere of Radius R

Trial wave function: $\Phi(r,R) := \frac{1}{\sqrt{2\cdot\pi\cdot R}}\cdot\frac{\sin\left(\frac{\pi\cdot r}{R}\right)}{r}$

Integral: $\int_0^{\infty} \blacksquare\cdot 4\cdot\pi\cdot r^2\,dr$

Kinetic energy operator: $T = -\frac{1}{2\cdot r}\cdot\frac{d^2}{dr^2}(r\cdot\blacksquare)$

Potential energy operatory: $V = -\frac{1}{r}$

Demonstrate the wave function is normalized. $\int_0^{R} \Phi(r,R)^2\cdot 4\cdot\pi\cdot r^2\,dr \rightarrow 1$

Set up the variational energy integral.

$$E(R) := \int_0^{R} \Phi(r,R)\cdot\left[-\frac{1}{2\cdot r}\cdot\frac{d^2}{dr^2}\left(r\cdot\Phi(r,R)\right)\right]\cdot 4\cdot\pi\cdot r^2\,dr + \int_0^{R} \Phi(r,R)\cdot -\frac{1}{r}\cdot\Phi(r,R)\cdot 4\cdot\pi\cdot r^2\,dr$$

Minimize the energy with respect to the variational parameter R.

$R := 1$ $\quad$ $R := \text{Minimize}(E,R)$ $\quad$ $R = 4.049$ $\quad$ $E(R) = -0.301$

The exact ground state energy for the hydrogen atom is -.5 E_h. Calculate the percent error.

$$\frac{-.5 - E(R)}{-.5} = 39.793\%$$

Compare optimized trial wave function with the exact solution by plotting the radial distribution functions.

$S(r) := \frac{1}{\sqrt{\pi}}\cdot\exp(-r)$ $\quad$ $r := 0, .02 .. 4.2$