Peer Instruction and Inquiry-Based Learning in Calculus I

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Abstract: We describe a format for a Calculus I course that attempts to incorporate two pedagogies: Peer Instruction (commonly associated with "clickers") and Inquiry-Based Learning. Additionally, Standards-Based Grading was employed to assess and grade the students. To do this, the course was accelerated so that course material was covered twice during the semester: once with Peer Instruction and once with Inquiry-Based Learning. We measured the effectiveness by measuring the change in performance on the Calculus Concept Inventory.

Keywords: Peer Instruction, PI, Inquiry-Based Learning, IBL, Calculus, Standards-Based Grading, SBG

1 INTRODUCTION

We will discuss the results of a teaching experiment in two Calculus I sections at a midwestern liberal arts college in the Fall of 2012. The sections had 20 and 26 students, totalling 46 students. This experiment was undertaken to solve several explicitly stated problems. The experiment involved two parts: abandoning lecture as a primary class pedagogy in favor of Peer Instruction and something akin to Inquiry-Based Learning, and abandoning the typical "points-based" grading system in favor of Standards-Based Grading. In order to accommodate these changes, the pacing of the course was adjusted in a non-traditional way. To determine the success of this experiment, the classes were given the Calculus Concept Inventory as a pre- and post-test. These results were compared against known results from other classes.

2 PROBLEMS TO BE SOLVED

Below are a list of problems surrounding teaching college-level courses. The experiment's goal was to help solve these problems. All but the first concern grading.

1. **Students do not learn as much as they could.** This problem is not particularly surprising, but it is important.
2. **Grades lack intrinsic meaning.** In the traditional points-based system, a student will receive an A for the course if and only if the student gets at least 90% on a weighted average of homework/quizzes/exams/etc. But defining grades in terms of points is problematic. For one, it simply moves the undefined term from "grade" to "points." Another problem is that defining grades in terms of points does not communicate expectations to the students well; telling a student that a 90% will yield an A does not tell the student what he should be doing to learn. The main point of grading is to communicate with students, other professors, the school, and employers. It is our opinion that the current points-based grading system is a poor tool for communication and needs improvement.
3. **Students are expected to learn the course content at the same pace.** It is clear that students learn at different rates. However, this is not recognized under the points-based grading system. Every student must complete each homework assignment/quiz/exam at the same time. The student's score is averaged with other assignments, so a poor performance on one assessment permanently lowers the student's grade. We think this is undesirable—our position is that artificial learning deadlines should be minimized.
4. **Grading does not give students feedback on how to improve.** A student who gets a 60% on a quiz knows that she did not do well. However, the score of "60%" does not give the student any direction on how to improve. Once again, we see that this points-based grading system does not do a good job at its task of communicating information about performance.

5. **Grading is too "high stakes."** It is conceivable that a student, due to illness, nerves, or personal problems, might perform poorly on a single test for reasons that do not reflect her understanding of the course material. This risks giving a grade that does not communicate the student's understanding.
6. **Cramming is rewarded.** A complementary problem to the previous one is that having few important assessments means that cramming can be a successful strategy for students if the number of assessments is few.
7. **There is not enough accountability to learning.** Underwood Dudley [2] said it best: "The chair of a department of a Big Ten university once observed, probably after a bad day, that it was possible for a student to graduate with a mathematics major without ever having solved a single problem correctly. Partial credit can go a long way." Under a points-based grading scheme, a calculus student could potentially earn an A in calculus classes without understanding the Fundamental Theorem of Calculus.

3 OVERVIEW OF METHODS

3.1 PEER INSTRUCTION

The calculus course was modified to help solve the above problems. In order to improve student learning, the tradional lecture was replaced with Peer Instruction (PI) and Inquiry-Based Learning (IBL). We first discuss the implementation of PI.

Peer Instruction, popularized by Eric Mazur (see, for instance, [6]), is a method of instruction that revolves around asking students a series of questions in class. Here is one such question from last semester:

At $(0, 0)$, the graph of $f(x) = x$

1. has no tangent line.
2. has exactly one tangent line.
3. has exactly two tangent lines.

4. has infinitely many tangent lines.

Students answered the questions using a classroom response system, more commonly known as "clickers." These are remote controls that allow the instructor to almost instantaneously see a histogram of the class's answers.

The students first answered individually. If the histogram showed the class overwhemingly got the question correct, we provided a short summary lecture and quickly moved on to the next question. If many students answered incorrectly, students discussed their answers in teams of three. After the team reached consensus, they re-voted.

The class results usually improved significantly after this discussion. If the class then overwhelmingly got the correct answer, we would give a brief lecture on why each answer was correct or not. Otherwise, we would give a more involved lecture.

To open up more time for Peer Instruction in class, this class was "flipped." That is, the transmission of information occurred outside of the classroom, and class time was spent making sense of the material. To do this, students read the textbook at home, answered some questions about the reading online the night before class, and then spent all of class working with questions similar to the one given above. Initially, there was concern that students would not do the work prior to class. However, there were actually few issues with students being unprepared for class.

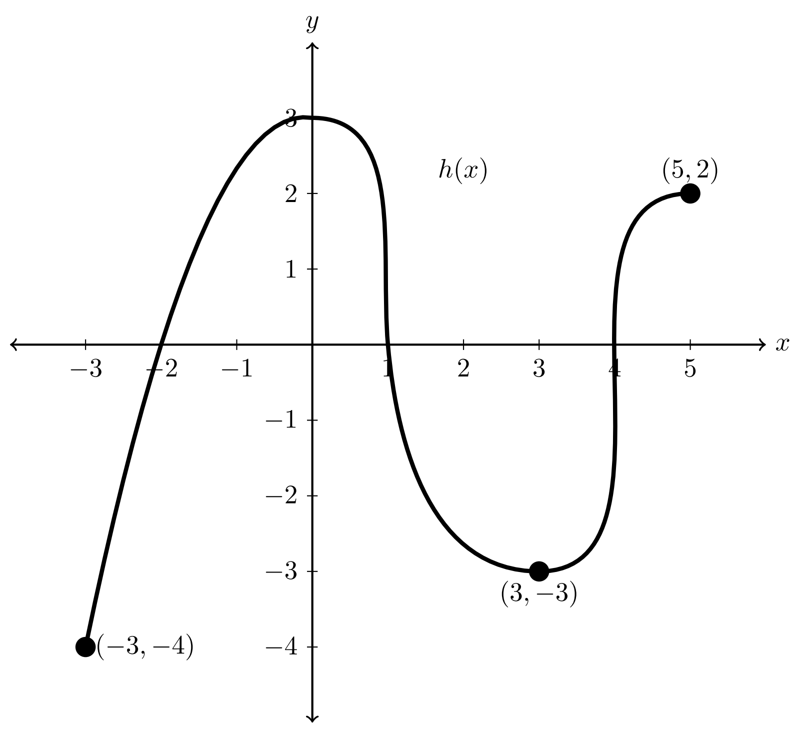
There is ample evidence from physics that Peer Instruction is superior to lecture, and similar work is starting to happen in mathematics. Please see [1] and [6] for more details.

3.2 INQUIRY-BASED LEARNING

The second pedagogy used to improve student learning was Inquiry-Based Learning (IBL). This is most identified with R.L. Moore and his "Moore Method," although this is only one variation of many. The definition of IBL is still under construction, but IBL typically involves giving students the primary responsibility for developing the mathematics, presenting the mathematics, and determining whether the mathematics is valid. An IBL class typically features student presentations and working in groups (IBL and PI each fall under the umbrella of "Interactive Engagement" [4], which roughly refers to a teaching method that involves active learning and immediate feedback).

In this calculus class, students were given a list of homework problems. The students worked on the problems at home, and then presented their results in class. Below are three such problems:

1. Find y' if $y = x^2 \cdot 2^x$.
2. Assume that f and g are differentiable. Differentiate $p(x)$ if $p(x) = \sqrt{g(x) + \ln f(x)}$.
3. In the graph of $h(x)$ is given below, for which values of x is $h'(x)$ positive?



The students were given roughly thirty of these questions at a time.

They were allowed to choose which problems they wanted to work on. If a student wanted to present a problem, the student submitted her preferences online before class. A presentation schedule was made prior to class and displayed at the beginning of class. Each student presentation lasted 2-5 minutes and was followed by questions from the class and the instructor.

There is also a growing body of evidence that IBL leads to improved student learning. See Laursen [5] and Christian Smith [7] for details.

3.3 STANDARDS-BASED GRADING

The usual method of grading relies heavily on chronology. That is, grades are organized in terms of "Quiz 5" and "Homework Set 3." Standards-Based Grading (SBG) instead organizes grading by content topic. For instance, here are the topics from this calculus class (some topics are repeated to give them a higher weight):

- Graphs of functions
- Shifting and scaling functions
- Trigonometry
- Limits
- Definition of Derivative
- Derivative—Symbolic
- Derivative—Symbolic
- Tangent Lines
- Second Derivatives and Concavity
- Linear Approximation
- Extrema
- Computing Integrals
- Computing Integrals
- The Fundamental Theorem of Calculus II

Of course, one can argue with this particular list of topics—it is definitely incomplete. However, traditional testing also only assesses a sample of the possible topics from the semester; Standards-Based Grading is simply systematic and transparent about the sampling (It was still possible to assess other topics in this class. For instance, the IBL-type presentations contained topics not listed here).

Most of the course grading was done through quizzes, which occurred 1-2 times per week. Each quiz question was explicitly tied to a topic.

Here is an example:

1. (Finding and classifying extrema) Find and classify the local extrema of $f(x) = x^{1/5}(x+1)$, and state where the function is increasing.

Quizzes were generally ten questions long, and students had 70 minutes to complete them. There was typically at most one question from each quiz topic.

The grading for each quiz question was binary: either a problem was completely correct and the student received credit, or there was a mistake and the student did not receive credit. The grade is then essentially a count of how many quiz questions are correctly answered for each topic. In particular, there is no penalty for getting any particular quiz question wrong; a student simply does not improve her grade by answering a question incorrectly.

The student's were tasked with answering four quiz questions for each topic correctly. When a student had correctly answered four questions on a given topic, she could skip questions on that topic for the remainder of the semester. This translated to a semester grade in the following way: a student received at least a C for the semester if and only if the student answered four quiz questions correctly in each topic . Whether a student received a C, B, A, or variation thereof was determined by averaging the student's grade for the IBL-type presentations and the student's grade for the final exam.

This system solved all of my problems with grading to some degree:

1. **Grades lack intrinsic meaning.** Now a grade of C means that a student has demonstrated procedural proficiency in the given topics.
2. **Students are expected to learn the course content at the same pace.** Students are expected to answer four questions for each topic by the end of the semester. A student who answers all of her questions correctly on four quizzes in September will get the same grade as a student who correctly answers all of his questions correctly on four quizzes in December.

3. **Grading does not give students feedback on how to improve.** A student immediately knows her strengths and weakness by seeing how many questions she has answered correctly for each topic. If she does not have many “Limits” question correct yet, she knows that she should improve her understanding of limits.
4. **Grading is too “high stakes.”** Students are never hurt by getting a question wrong. In this sense, a student’s grade is non-decreasing. This likely has some psychological benefits.
5. **Cramming is rewarded.** Quizzes occur 1-2 times per week, and it is rare to have more than one question occur per topic on a quiz. Consequently, a student could only be successful in a topic if he understood the material well enough on at least four different quizzes. Thus, students need to demonstrate sustained knowledge of each topic.
6. **There is not enough accountability to learning.** Under a traditional grading scheme, students can “hide” difficiencies by getting partial credit on their problem areas and doing well on the other questions. In SBG, students must learn every single topic to get a C; a student who never quite gets the Fundamental Theorem of Calculus will get a grade lower than a C.

Anecdotally, this system had one other advantage. In the first week of class, four students approached me with concerns about their algebra background; they were worried they did not have the skills to do well in the class. A pre-test confirmed that these four students struggled with algebra and trigonometry, and my previous experience as a calculus instructor led me to believe that there was not much hope for them to do well. However, all four of these students ended up fulfilling the quiz requirement, thereby guaranteeing each of them at least a C for the course. In addition to the students’ hard work and support from the teaching staff, this appears to be due to the fact the material they needed to master was clearly communicated by SBG, the students were unambiguously required to learn the material under SBG, and the students had multiple chances to be successful under SBG. Moreover, this has previously occurred in multiple semesters in several different classes—students whom it would be easy to "give up on" early in the semester end up doing surprisingly well under SBG.

4 INTEGRATION OF METHODS

Our goal is to integrate these two pedagogies in a way that allows us to use the grading scheme described above. We did this by splitting the semester into two parts: the first portion would use Peer Instruction and cover the entire content of the course quickly, and the second portion would use IBL-type presentations to re-do all of the course content. Part of the theory is that students will benefit from seeing course material twice—it might take time for some ideas to "sink in." A second advantage is that the Presentation portion of the course could be tailored to address the topics where the students struggled in the PI portion of the course. Finally, exposure to all of the content in the first half of the semester meant that every topic from the SBG grading scheme could be tested for the entirety of the second half of the semester, giving students ample time to demonstrate proficiency in each quiz topic.

5 RESULTS

Student achievement was measured using the Calculus Concept Inventory (CCI) [3], a test that measures conceptual understanding of calculus. The CCI was modelled after the Force Concept Inventory from physics, and the techniques for measuring student improvement are similar.

The students were given the test in the first week of class as a pre-test and the last week of class as a post-test. Let x_{pre} be the course average (as a percentage) for the pre-test and x_{post} be the course average for the post-test. A "normalized gain" was calculated by the following formula:

$$\frac{x_{\text{post}} - x_{\text{pre}}}{100\% - x_{\text{pre}}}.$$

This normalized gain measures the amount of improvement for a class. So a normalized gain of 0.37 means that students, on average, learned about 37% of the material they did not already know. Lecture-based classes typically have a normalize gain between 0.15 and 0.23, whereas "Interactive Engagement" classes such as those studied in the Fall of 2012 can be around 0.3 [3]. However, the studied sections actually had normalized gains of 0.2075 and 0.1440; pooling the two sections resulted in a normalized gain of 0.1762. This is comparable to a lower-performing lecture.

6 DISCUSSION AND CONCLUSION

The normalized gain results were certainly disappointing. However, we have a good hypothesis as to why this class did not have results as good as other "Interactive Engagement" classes: the Presentation portion of the class was not run well.

There were two main problems with the Presentation portion of the course. First, I have come to realize that—while the classes superficially looked like IBL classes—the Presentation portion of the class is not actually IBL. While the students presented the mathematics, they did little to develop or validate the mathematics, which are likely where most of the learning occurs in an IBL class. Thus, we should not have expected to see the same gains experienced in other IBL courses. Second, the general format of the Presentation classes was that students choose the problems that they will attempt, they solve them, and then they present the solutions in class. The problem with this method is that, since each student chose which problems she would attempt, most of the audience had not thought about a particular problem before seeing the solution. *The effect of this was that the last half of the semester was not only essentially lecture-based, but the lectures were given by non-experts—the other students.* With this in mind, it is not surprising that this class's results were more similar to lecture-based classes than "Interactive Engagement" classes—it is reasonable to think that the novice-led lectures undid some of the gains achieved from the Peer Instruction portion of the course.

There are several things that could be done to improve the Presentation portion of the course, even without converting to a full-fledged IBL class. For instance, we could reduce the number of problems available to the students and require that each student attempt all of the problems. This would mean that all students will have thought about the presentation material prior to seeing the presentation. A second idea is to have students discuss some of the problems in small groups prior to the presentations. Each of these suggestions would help student to get more from the presentations. A third suggestion is to focus less on the presentations and spend more of class time working on problems.

Another factor that might explain the low normalized gain score is the grading system. In order to get at least a C for the course, students had to demonstrate procedural knowledge on quizzes. However, the Calculus Concept Inventory measures *conceptual* knowledge. An easy fix would be to make quiz categories that measure conceptual understanding. This would lead students to focus on conceptual understanding throughout the semester, which may lead to better results on the Calculus Concept Inventory. Regardless, this matches our goals for the course more closely anyway, and so future courses will have assessment focus more on conceptual understanding.

In spite of the low normalized gain, we feel this method is worth trying again. This is partially due to anecdotal data—we have tried this course format, seemingly successfully, in other courses (real analysis, complex analysis, and multivariable calculus). However, we do not have any data for the other courses. For calculus, we have identified a problem area—the Presentation classes—and hope to improve upon that aspect. This, combined with changing the focus of the quizzes, may lead to improvements in the conceptual understanding of students.

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BIOGRAPHICAL SKETCH

Bret Benesh teaches mathematics at the College of Saint Benedict and Saint John's University in Central Minnesota. He has taught at the University of Wisconsin-Madison as a graduate student and later at Harvard University. His professional interests include finite group theory and preparing pre-service elementary education majors. He has a wonderful wife and two adorable children, only one of whom is named after a mathematical function.