Problems that combine conservative and non-conservative forces

YF Chapter 7, Section 3, page 196

All of the problems in Chapter 7 can be done using a combination of the work-energy theorem and the ideas of potential and kinetic energy—the book's approach on page 196 strikes me as unnecessarily complicated. In particular, the definition of "internal energy" is (to me, at least) not very clear.

In this respect, compare Example 7.5, which shows the way I work these problems, with Example 7.12, which uses the notion of "internal energy." Here is the way I would work this example—it is almost the same as the book's Example 7.5. As the mass moves from point 1 to point 2 ,

- It gains kinetic energy;
- It loses potential energy; and
- it also loses mechanical energy to friction. This friction is converted into heat, which will be absorbed by the mass itself, or by its surroundings.

Under these circumstances, we can say by the work-energy theorem that

$$\Delta KE = W_{grav} + W_{friction}$$

We want to find the work done by friction (a *non-conservative* force), given the information in the problem:

$$W_{friction} = \Delta KE - W_{grav}$$

We can calculate the work done by gravity in two ways:

First method: We note that the work done is negative, since the force and the displacement are in opposite directions. Hence $W_{grav} = -mg\Delta h$, where Δh is the absolute value of the change in height.

Second method: Gravity is a conservative force. Hence we can use the change in potential energy to find the work. The potential energy function *U* is given by $U = mgh$; hence

$$\Delta U = mg(h_f - h_i) \quad \text{and}$$
$$W_{grav} = -\Delta U = -mg(h_f - h_i) = -mg\Delta h$$

The result is the same with both methods! Now to the numbers; note that we take $y = 0$ at the bottom of the circular arc.

$$\Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 450 \text{ J} - 0$$
$$\text{W}_{\text{grav}} = -\Delta U = -\left(U_f - U_i\right) = -\left(0 - MgR\right) = +735 \text{ J}$$

Note that the work done is positive, and the change in potential energy is negative. Thus, part of the initial potential energy (*MgR*) has gone to increase the kinetic energy, and the rest is lost to frictional work. We can calculate that frictional work:

$$W_{friction} = \Delta KE + \Delta U = \Delta KE - W_{grav} = 450\ \text{J} - 735\ \text{J} = -285\ \text{J}$$

This frictional work is of course negative—the force and displacement are in opposite directions. Thus, the effect of friction is to decrease the total mechanical energy.

In Example 7.12, the text carries matters one step farther—it asks what has happened to that lost frictional work: It reappears in the form of heat, or “thermal energy.” Thus, the text defines an “thermal energy” ΔU_{int} that is just the negative of the work done by friction. That definition is unnecessary if all we are concerned with is the mechanical energy of the person.