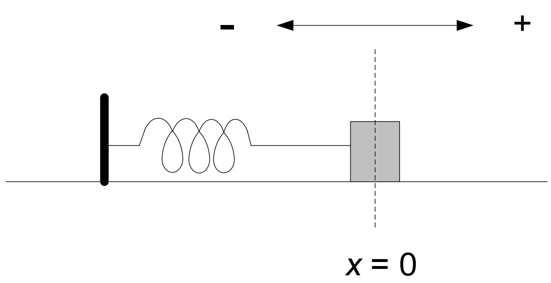
Hooke's Law force

Consider a spring exerting a force on a mass. Suppose the mass is on a frictionless horizontal surface.



Experiment shows that the force that the spring exerts on the mass is proportional to the distance *x* that the spring stretches from its equilibrium position.

$$F = -kx$$

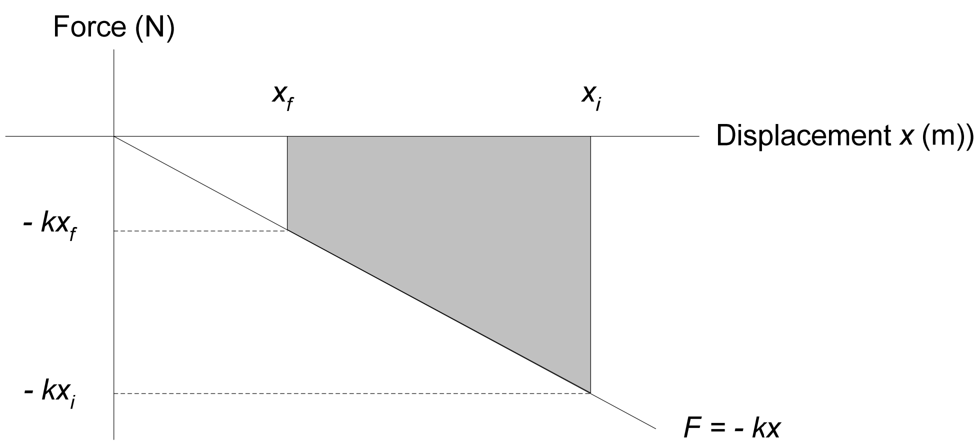
We call *k* the spring constant. Note that it has units N/m.

The negative sign means that the force on the mass tends to restore the mass to its equilibrium position. For example, if we pull the mass to the right ($x > 0$), then the spring exerts a force to the left ($F < 0$). Likewise, if we push the mass to the left ($x < 0$), the spring exerts a force on the mass to the right ($F > 0$). We call such a force a **restoring force**.

YF takes this approach later in the book—see for example Fig. 7.13 (page 223) and the Chapter 7 summary (page 237).

In Chapter 6, however, YF calculate the force the mass exerts on the spring. It is easy to see that this force is just the negative of mine—it's an example of Newton's third law. See the CAUTION in YF, page 194. Both are correct, buy my approach is the more usual one in physics textbooks. In addition, it seems to me that my approach makes it easier to think about the Work-Energy Theorem.

As we showed in class, the work done **by the spring on the mass** is represented by the gray area under the curve:



Suppose the mass is initially at stretched to the position $x_i > 0$ and released. The mass accelerates to the left, and its speed increases when it has reached some final position x_f: The work is positive. Let's calculate it:

$$W = \int_{x_i}^{x_f} -kx\,dx = -\frac{1}{2}kx^2\Big|_{x_i}^{x_f}$$
$$= -\frac{1}{2}kx_f^2 + \frac{1}{2}kx_i^2$$

Note that the sign of this result is just the opposite of YF Eq. 6.10, as it should be. Remember that they are calculating the work **done by the mass on the spring.**

Note also that this work is positive if $x_i > x_f$, as it is in our example.

Exercise: Confirm this result by calculating the difference in the areas of the large and small triangles, as we did in class.

Work-Energy Theorem

We can now state the Work-Energy theorem: The work done **on the mass** by the spring is equal to the change in kinetic energy **of the mass**:

$$W = KE_f - KE_i$$
$$-\frac{1}{2}kx_f^2 + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Exercise: Show that for the example given above ($x_i > x_f$), the work is positive, and hence the mass is speeding up.

Exercise: Look at a couple of other problems, and show that in all cases, the signs of the work and the change in kinetic energy are given correctly by this equation.

Exercise: Note that in Example 6.6, the book is calculating the work done **on the spring**, and hence the sign agrees with Eq. 6.10. Again, note the CAUTION on page 194.