PROBLEMS
CLASS 3
Chapters 1 and 2

Exercise 2.39

Let's choose x = 0 at ground level, and positive up. Our equations for the flea are

$$x = v_0 t - \frac{1}{2} g t^2$$
$$v = v_0 - gt$$
$$v^2 = v_0^2 - 2gx$$

(a) It is convenient to use the last equation. Since v = 0 at the highest point, we have

$$x := 0.44\,m \qquad v_0 := \sqrt{2 \cdot g \cdot x} \qquad v_0 = 2.94\,m\,sec^{-1}$$

(b) From the second equation, again since v = 0 at the highest point,

$$t := \frac{v_0}{g} \qquad t = 0.3\,sec$$

is the time it takes to get to the highest point. But it takes the same time to fall back to the ground (why?). Therefore the total time in the air is 0.6 sec.

Problem 2.83

Choose positive up.

Initially, the ball is accelerated upwards as it moves 0.64 m up. We start at x = 0, and at zero initial velocity, so we have

$$v^2 = 2ax$$

(a) Hence

$$x := 0.64\,m \qquad a := 45 \cdot \frac{m}{sec^2}$$

$$v := \sqrt{2 \cdot a \cdot x} \qquad v = 7.589\,m\,sec^{-1}$$

(b) This value will be the initial velocity for the free-fall part of the problem. The shot is launched at this speed, 2.2 m above the ground. We choose ground level to be x = 0, so the equations describing the motion are

$$x = x_0 + v_0 t - \frac{1}{2} g t^2$$
$$v = v_0 - gt$$
$$v^2 = v_0^2 - 2g\left(x - x_0\right)$$

where

$$v_0 := 7.589 \frac{m}{sec} \qquad x_0 := 2.2 \cdot m$$

We use the second equation to find how long it takes to reach maximum height ($v = 0$), and then use the first equation to find the maximum height:

$$t := \frac{v_0}{g} \qquad t = 0.774\text{sec}$$

$$x := x_0 + v_0 \cdot t - \frac{1}{2} \cdot g \cdot t^2 \qquad x = 5.14\text{m}$$

(c) We find the time it takes to fall from rest, at height 5.14 m, to $x = 1.83$ m. The equation is

$$x = x_0 - \frac{1}{2} g t^2$$

$$1.83 = 5.14 - \frac{1}{2} g t^2$$

$$t := \sqrt{\frac{2 \cdot (5.14\text{m} - 1.83\text{m})}{g}} \qquad t = 0.822\text{sec}$$

We need the time up + time down:

total_time := 0.774·sec + 0.822·sec   total_time = 1.60sec

Problem 2.87

Choose the origin at the top of the building, and choose positive down. Let h be the height of the building, and t be the time it takes for the object to fall to the ground, starting from rest. The two conditions given by the problem may be written:

$$h = \frac{1}{2} g t^2 \quad \text{and} \quad \frac{3}{4} h = \frac{1}{2} g (t-1)^2$$

(Be sure you understand the second equation: Since the last ¼ h takes 1 sec, the first ¾ h must take $(t-1)^2$. Multiply the second equation by 4/3, so that one can equate the two right-hand sides, and solve for the time.

$$h = \frac{1}{2} g t^2 \quad \text{and} \quad h = \frac{4}{3} \frac{1}{2} g (t-1)^2$$

$$\frac{1}{2} g t^2 = \frac{4}{3} \frac{1}{2} g (t-1)^2$$

$$t^2 = \frac{4}{3} (t-1)^2$$

Notice that there has been some cancellation in the last step—it's another reason to leave the arithmetic until the end, as much as possible. Let's continue with the algebra:

$$t^2 = \frac{4}{3}(t-1)^2 = \frac{4}{3}(t^2 - 2t + 1)$$

$$3t^2 = 4(t^2 - 2t + 1)$$

$$0 = t^2 - 8t + 4$$

We solve the quadratic equation:

$$t = \frac{8 \pm \sqrt{64-16}}{2} = 7.46 \text{ sec}, \, 0.536 \text{ sec}$$

The second solution is spurious (why?). We can now substitute the time into either equation to find the height.

$$h = \frac{1}{2} g t^2 = \frac{1}{2}(9.8)(7.46)^2 = 273 \text{ m}$$

HERE IS A SECOND, MORE CUMBERSOME SOLUTION:

Let the top of the building be $x = 0$, and choose positive down, so that ground level is $x = h$. We first find the velocity after the object has fallen from rest to $x = \frac{3}{4} h$.

$$\begin{aligned} v^2 &= 2gx \\ v^2 &= 2g\frac{3h}{4} = \frac{3}{2} gh \\ v &= 3.835\sqrt{h} \end{aligned}$$

In this problem, it will probably be easier to put in the numbers early. We use this initial velocity to write an equation for the last ¼ h of the fall. Adopt a new coordinate system with $x = 0$ at $h/4$ above the ground. The object has the initial velocity already calculated. So we have

$$\begin{aligned} x &= v_0 t + \frac{1}{2} g t^2 \quad \text{(for positive down)} \\ x &= 3.835\sqrt{h}\, t + 4.9t^2 \end{aligned}$$

But we know $x = h/4$ at $t = 1$ sec. Therefore

$$\begin{aligned} x &= 3.835\sqrt{h}\, t + 4.9t^2 \\ \frac{h}{4} &= 3.835\sqrt{h} + 4.9 \\ h &= 15.34\sqrt{h} + 19.6 \\ h - 19.6 &= 15.34\sqrt{h} \\ \left(h - 19.6\right)^2 &= 235.32\, h \end{aligned}$$

Thus we have reduced the problem to a quadratic equation in h. We have a little more algebra to do:

$$\begin{aligned} \left(h - 19.6\right)^2 &= 235.32\, h \\ h^2 - 39.2h + 384.16 &= 235.32\, h \\ h^2 - 274.5h + 384.2 &= 0 \end{aligned}$$

$a := 1 \qquad b := -274.5 \qquad c := 384.2$

$$h_1 := \frac{-b + \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \qquad h_1 = 273 \ \text{m}$$

$$h_2 := \frac{-b - \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \qquad h_2 = 1.41 \ \text{m}$$

The second result is clearly spurious. The building is about 273 meters high.

Chapter 1, Question Q1.18

If the lengths of the vectors A and B add to give the length of C, then A and B must lie in the same direction; otherwise $A + B < C$. If $C = 0$, then A and B must be equal in length, and be in opposite directions (that is, 180 degrees apart). See if you can illustrate these results with drawings.

Exercise 1.39

From the figure, we have

$$A_x = 0 \qquad A_y = -8.00\,\hat{j}$$

$$B_x = 15.0\cos 60^\circ \qquad B_y = 15.0\sin 60^\circ$$
$$B_x = 7.5 \qquad B_y = 13.0$$

To find the various vectors, we just add the components:

$$\vec{A} + \vec{B} = -8\hat{j} + 7.5\hat{i} + 13\hat{j}$$
$$= 7.5\,\hat{i} + 5\hat{j}$$

We find magnitude and direction in the usual way: If $\vec{C} = \vec{A} + \vec{B}$,

$C_x := 7.5 \qquad C_y := 5$

$$C := \sqrt{{C_x}^2 + {C_y}^2} \qquad C = 9.01$$

$$\theta := \operatorname{atan}\left(\frac{C_y}{C_x}\right) \qquad \theta = 33.69\text{deg}$$

Clearly $\vec{A} + \vec{B} = \vec{B} + \vec{A}$. The other vectors work out similarly:

$$\vec{A}-\vec{B} = -8\hat{j} - \left(7.5\hat{i} + 13\hat{j}\right)$$
$$= -7.5\,\hat{i} - 21\hat{j}$$
$$\vec{B}-\vec{A} = 7.5\,\hat{i} + 21\hat{j}$$

The magnitudes will be the same; if $\vec{D} = \vec{A} - \vec{B}$,

$D_x := 7.5$ $\quad$ $D_y := 21.0$

$D := \sqrt{D_x^{\,2} + D_y^{\,2}}$ $\quad$ $D = 22.3$

$\theta := \operatorname{atan}\left(\frac{D_y}{D_x}\right)$ $\quad$ $\theta = 70.3\text{deg}$

The angles require interpretation; the first vector is in the third quadrant, is really $\theta + \pi = 250.3^o$; the second vector is in the first quadrant, so the calculated value is correct.

Exercise 1.49

We proceed in the same way as above:

$C := \sqrt{C_x + C_y^{\,2}}$ $\quad$ $C = 5.70$

$\theta := \operatorname{atan}\left(\frac{C_y}{C_x}\right)$ $\quad$ $\theta = 33.69\text{deg}$

Note that for ***B*** we have measured the angle from the positive x axis. We want the vector $\vec{C} = 3\vec{A} - 4\vec{B}$. We begin by adding components:

$D_x := -7.5$ $\quad$ $D_y := 2.1$

Or in other words, $\vec{C} = 12.0\,\hat{i} + 14.9\hat{j}$. It is straightforward to convert to magnitude and direction:

$C := \sqrt{C_x^{\,2} + C_y^{\,2}}$ $\quad$ $C = 19.17$

$\theta := \operatorname{atan}\left(\frac{C_y}{C_x}\right)$ $\quad$ $\theta = 51.2\text{deg}$

Problem 1.68

(Note that the unit of force used in the statement of the problem is the Newton, abbreviated N. We will encounter this unit later on; don't worry about it for now.)

We require a new vector $\vec{D}$ such that

$$\vec{A} + \vec{B} + \vec{C} + \vec{D} = 0 \quad \text{or}$$

$$\vec{D} = -\left(\vec{A} + \vec{B} + \vec{C}\right)$$

We first resolve the three given vectors into components, and then calculate the components of $\vec{D}$. Remember that we always measure the angle from the positive *x* axis.

$A := 100 \qquad B := 80 \qquad C := 40$

$A_x := A \cdot \cos(30 \cdot deg) \qquad A_y := A \cdot \sin(30 \cdot deg) \qquad A_x = 86.603 \qquad A_y = 50.000$

$B_x := 80 \cdot \cos(120 \cdot deg) \qquad B_y := B \cdot \sin(120 \cdot deg) \qquad B_x = -40.000 \qquad B_y = 69.282$

$C_x := C \cdot \cos(233 \cdot deg) \qquad C_y := C \cdot \sin(233 \cdot deg) \qquad C_x = -24.073 \qquad C_y = -31.945$

$D_x := -\left(A_x + B_x + C_x\right) \qquad D_y := -\left(A_y + B_y + C_y\right) \qquad D_x = -22.53 \qquad D_y = -87.34$

Or in other words,

$$\vec{D} = -22.53\,\hat{i} - 87.34\,\hat{j}$$

The problem asks us to give an answer in terms of magnitude and direction.

$D := \sqrt{D_x^{\,2} + D_y^{\,2}} \qquad D = 90.2$

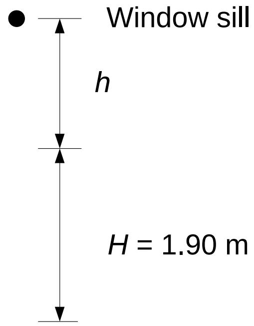
$\theta := \text{atan}\left(\dfrac{D_y}{D_x}\right) \qquad \theta = 75.535 deg$

But this angle is in the first quadrant, and it's apparent that our vector is in the fourth quadrant; hence the actual angle is

$\theta + 180 \cdot deg = 255.5 deg$

Extra Credit: Problem 2.80

A diagram may help here:



Our goal is to find the unknown distance *h* from the upper window sill to the top of the lower window.

It is convenient to divide the problem into two parts:

Part 1: Choose $x = 0$ at the top of the lower window, and choose positive down. Our equations are

$$x = v_0 t + \frac{1}{2} g t^2$$
$$v = v_0 + gt$$

Use the first equation: When the pot is at the bottom of the lower window, $x = 1.90\ \text{m}$ and $t = 0.420\ \text{s}$. The only thing we don't know is v_0. We can solve it to find

$$x := 1.90 \cdot \text{m} \qquad t := 0.420 \text{sec}$$

$$v_0 := \frac{x - \frac{1}{2} \cdot g \cdot t^2}{t} \qquad v_0 = 2.46 \text{m sec}^{-1}$$

Part 2: Here we choose $x = 0$ to be the upper window sill from which the flower pot falls. We know its velocity when it reaches the top of the lower window sill. Again take positive down. The pot falls, from rest (i.e., no initial velocity), so our equations are

$$v = gt$$
$$x = \frac{1}{2} g t^2$$

From the first equation we can find the time the pot takes to fall to the top of the lower sill:

$$v := 2.46 \cdot \frac{\text{m}}{\text{sec}} \qquad t := \frac{v}{g} \qquad t = 0.251 \text{sec}$$

We can use the first equation to find the distance *h* that the pot falls in this time:

$$h := \frac{1}{2} \cdot g \cdot t^2 \qquad h = 0.31 \text{m}$$