Problems
Class 28
Chapters 10, 11

Problem 10.96

Suppose the turntable is rotating CCW; if we call CCW positive, its initial angular momentum is $L_{ti} = I\omega_i$. The runner is moving in a CW circle of radius *R*, where *R* is the radius of the turntable. Hence the runner's angular momentum is $L_{ri} = -mv_i R$; the sign indicates the direction of rotation. We set the sum equal to the final angular momentum:

$$L_i = L_f$$
$$I_t\omega_i - Mv_i R = I_f\omega_f$$

But the final moment of inertia is

$$I_f = I_t + MR^2$$

Hence we have

$$L_i = L_f$$
$$I_t\omega_i - Mv_i R = \left(I_t + MR^2\right)\omega_f$$

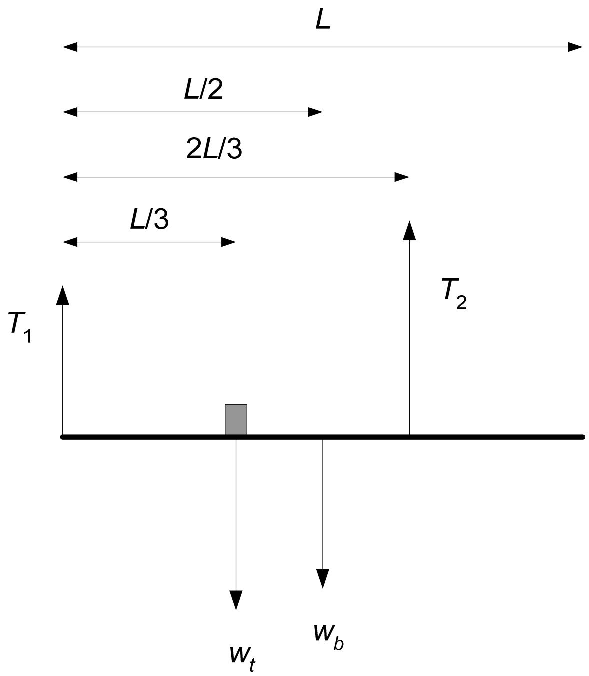
and solving for the angular velocity,

$I_t := 80 \cdot kg \cdot m^2$ $\quad \omega_i := 0.2 \cdot \frac{rad}{sec}$ $\quad M := 55 \cdot kg$ $\quad R := 3 \cdot m$ $\quad v_i := 2.8 \cdot \frac{m}{sec}$

$\omega_f := \frac{I_t \cdot \omega_i - M \cdot v_i \cdot R}{I_t + M \cdot R^2}$ $\quad \omega_f = -0.78 \frac{rad}{sec}$

The negative sign means that the turntable has reversed direction and is now rotating CW.

Exercise 11.8



The force diagram is shown on the left. First we set the forces equal to zero:

$$T_1 + T_2 - w_t - w_b = 0$$

Next we set the torques to zero. I will calculate the torque about the left end of the bar.

$$-w_t \frac{L}{3} - w_b \frac{L}{2} + T_2 \frac{2L}{3} = 0$$

We solve the second equation for T_2, and then find T_1 from the first equation:

$w_t := 25 \cdot N \qquad w_b := 50 \cdot N$

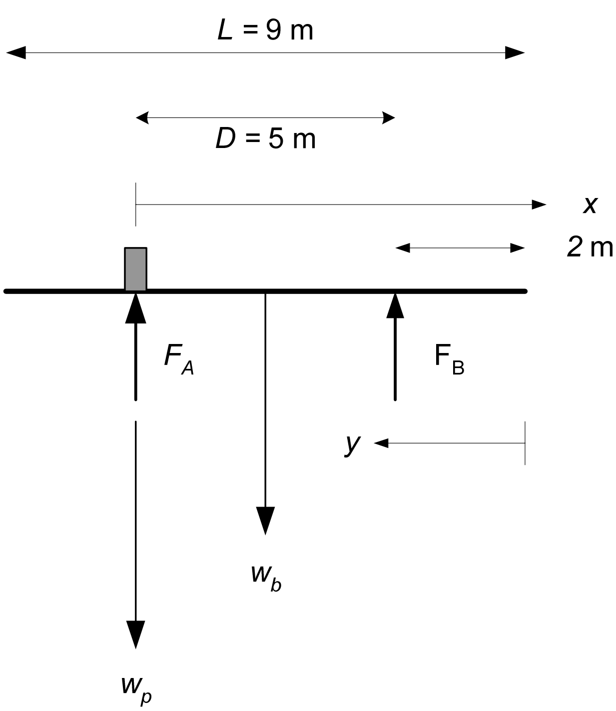
$T_2 := \frac{w_t}{2} + \frac{3}{4} \cdot w_b \qquad T_2 = 50\,N$

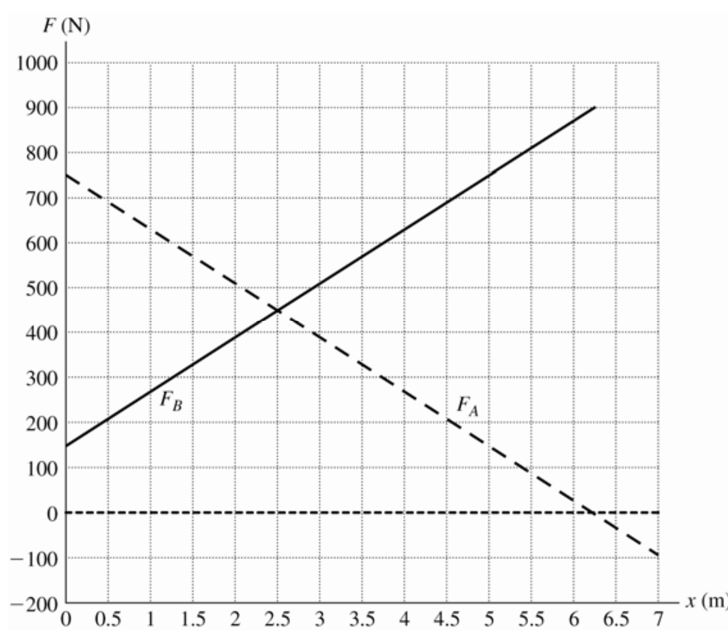
$T_1 := w_t + w_b - T_2 \qquad T_1 = 25\,N$

Note that one does not need the length of the bar or the lengths of the cables.

Exercise 11.12

The force diagram and the graph are as shown.





The force diagram suggests an approach to the graph. For convenience, the mass of the person is shown at x = 0. Note that when the person is in this position, right above the point at which F_A is applied, we must have

$$F_A = w_p + \frac{w_b}{2} = 600\ \text{N} + 150\ \text{N} = 750\ \text{N}.$$

$$\text{and hence } F_B = 150\ \text{N}$$

Likewise, when the person is right above the point at which F_B is applied, we must likewise have $F_B = 750\ \text{N}$ and $F_A = 150\ \text{N}$. If you're not convinced, set the torque about the point at which F_A is applied equal to zero, when is at x = 0, as shown in the force diagram:

$$-w_b \frac{D}{2} + F_B D = 0 \quad \Rightarrow \quad F_B = \frac{w_b}{2} = 150\ \text{N}$$

That information is enough to make the graph, though it is also easy to write a torque equation showing that when the boy is at the center of the beam, $F_A = F_B$, confirming the picture in the graph.

(b) If the graph has been done carefully, one sees that $F_A = 0$ when the person is at x = 6.25 m, or 1.25 m to the right of the support at point *B*.

(c) Now imagine the person standing at the right end of the beam in the force diagram. The force F_B is now applied at an unknown distance x from the origin (the point at which F_A is applied). Our condition is that the beam is just on the verge of tipping, so that $F_A = 0$, following the same reasoning as in Example 11.1. The force and torque equations, calculating the torque around the right end of the beam are:

$$F_A + F_B - w_b - w_p = F_B - w_b - w_p = 0 \quad \text{or}$$

$$F_B = w_b + w_p = 900\ \text{N}$$

$$-w_B \frac{L}{2} + F_B y = 0$$

where y is the distance of the new support point for *B* from the right end of the beam (see diagram). Substituting numbers,

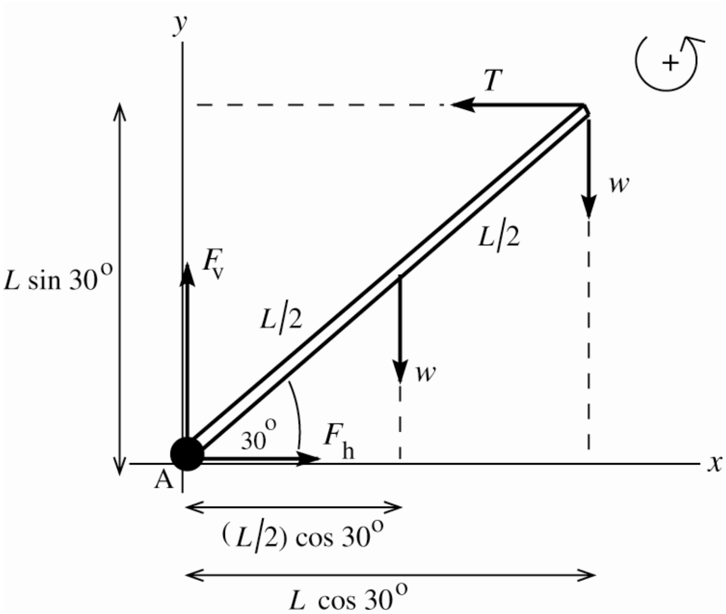
$$w_b := 300 \cdot \text{N} \qquad L := 9 \cdot \text{m} \qquad F_B := 900 \cdot \text{N}$$

$$y := \frac{L \cdot w_b}{2 \cdot F_B} \qquad y = 1.5\text{m}$$

that is, 1.5 m from the end of the beam, or 0.5 m to the right of its original position.

Exercise 11.13

(a) The force diagram is shown below:



We write the vertical and horizontal force equations, and the torque equation. I have calculated the torque about an axis through the pivot on the wall, so that the forces exerted by the pivot exert zero torque:

$$\text{vertical} \qquad \text{horizontal}$$
$$F_v - 2w = 0 \qquad F_h - T = 0$$
$$\text{torque}$$
$$-w\left(\frac{L}{2}\right)\cos 30^\circ - wL\cos 30^\circ + TL\sin 30^\circ = 0$$

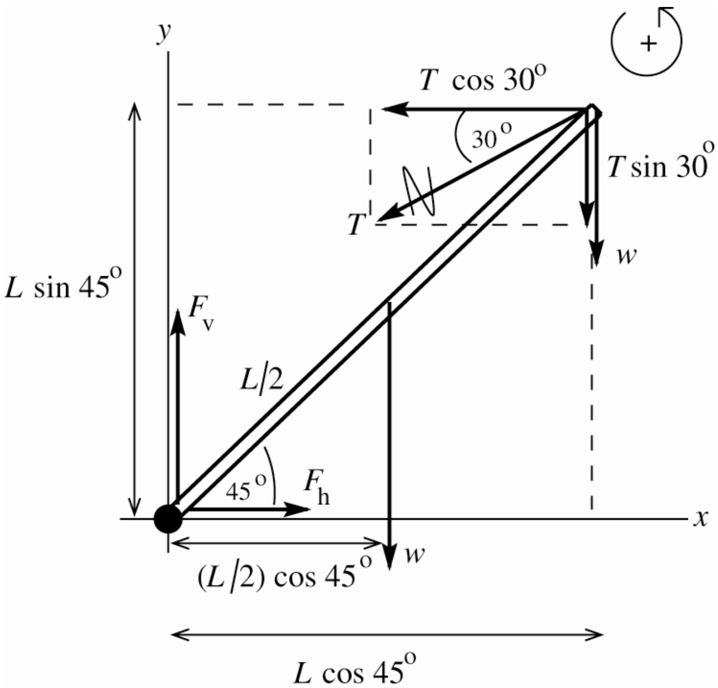
We solve the torque equation for *T*, and then find the vertical and horizontal components of the force exerted by the pivot.

$$T = \frac{3w}{2}\frac{\cos 30^\circ}{\sin 30^\circ} = \frac{3}{2}w\cot 30^\circ = 2.60w$$
$$F_c = 2w$$
$$F_h = T = 2.60w$$

Finally, the magnitude and direction of the force exerted by the pivot are

$$F_p = \sqrt{\left(2w\right)^2 + \left(2.60w\right)^2} = 3.28w$$
$$\theta = \tan^{-1}\frac{F_v}{F_h} = \frac{2}{2.6} = 0.769 \quad \Rightarrow \theta = 37.6^\circ$$

(b) The force diagram for this part is a little more complicated; but it can be simplified by resolving the tension into components:



We proceed much as before, writing the force and torque equations:

vertical

$$F_v - 2w - T\sin 30^\circ = 0$$

horizontal

$$F_h - T\cos 30^\circ = 0$$

torque

$$-w\left(\frac{L}{2}\cos 45^\circ\right) - w\left(L\cos 45^\circ\right) - \left(T\sin 30^\circ\right)\left(L\cos 45^\circ\right) + \left(T\cos 30^\circ\right)\left(L\sin 45^\circ\right) = 0$$

$$-\frac{w}{2} - w - T\sin 30^\circ + T\cos 30^\circ = 0 \quad \text{since } \sin 45^\circ = \cos 45^\circ$$

We proceed as before, solving the torque equation for *T* and then finding the components of the force exerted by the pivot.

$$T = \frac{\frac{3}{2}w}{\cos 30^\circ - \sin 30^\circ} = 4.1w$$

$$F_h = T\cos 30^\circ = 3.55w$$

$$F_v = 2w + T\sin 30^\circ = 4.049w$$

We find the magnitude and direction as before:

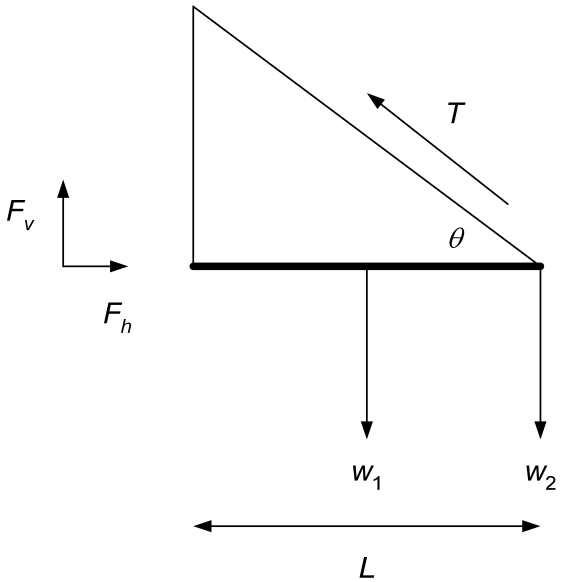
$$F_p = \sqrt{\left(3.55w\right)^2 + \left(4.049w\right)^2} = 5.38w$$

$$\theta = \tan^{-1}\frac{F_v}{F_h} = \frac{4.049}{3.55} = 1.14 \quad \Rightarrow \theta = 48.8^\circ$$

Aside: We could also calculate the torque about the right end of the bar. The torque equation turns out to be simpler, but the subsequent algebra a little more complex. It's a matter of taste.

Exercise 11.14

The force diagram is shown below.



As usual, we write horizontal and vertical force equations, and a torque equation:

horizontal

$$-F_h + T\cos\theta = 0$$

vertical

$$F_v + T\sin\theta - w_1 - w_2 = 0$$

torque

$$\left(T\sin\theta\right)L - w_1\frac{L}{2} - w_2 L = 0$$

We find the angle (3-4-5 triangle) solve the torque equation for *T*, and use the force equation to find the components of the force at the pivot:

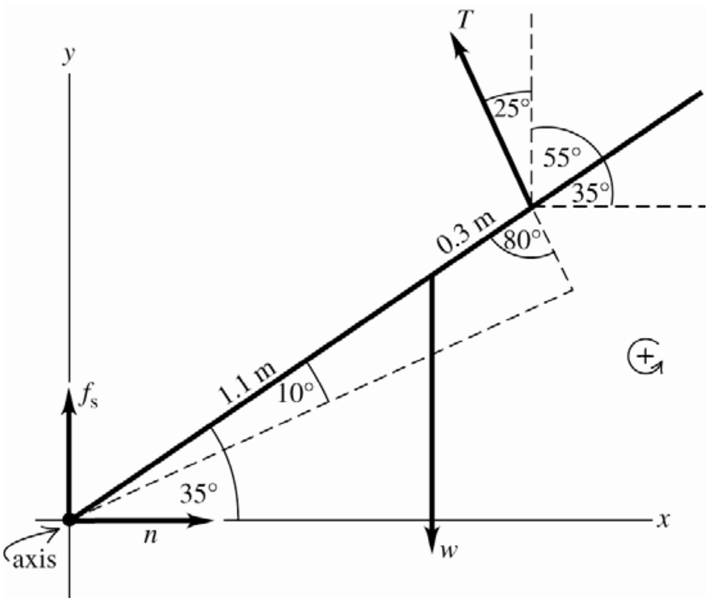
$\theta := \operatorname{asin}\left(\frac{3}{5}\right)$ $\theta = 36.87\text{deg}$ $w_1 := 150 \cdot N$ $w_2 := 300 \cdot N$

$T := \dfrac{\frac{w_1}{2} + w_2}{\sin(\theta)}$ $T = 625\,N$

$F_h := T \cdot \cos(\theta)$ $F_h = 500\,N$

$F_v := w_1 + w_2 - T \cdot \sin(\theta)$ $F_v = 75\,N$

Problem 11.41



The force diagram is moderately complicated—be sure that you understand it.

Note in particular that the tension *T* makes an 80° angle with the climber; the perpendicular moment arm—a line perpendicular to the line of action of the force—is therefore the longer dashed line shown in the figure.

We write the horizontal and vertical forces, and the torque equation:

$$\text{horizontal}$$
$$-n + T\sin 25^\circ = 0$$
$$\text{vertical}$$
$$f_s + T\cos 25^\circ - w = 0$$

For the torque equation, let *a* = 1.1 m and *b* = 0.3 m (see diagram). The moment arm perpendicular to *T* (longer dashed line) is $(a+b)\cos 10^\circ$.

$$\text{torque}$$
$$T\left(\left[a+b\right]\cos 10^\circ\right) - w\left(a\cos 35^\circ\right) = 0$$

(a) The tension is

$a := 1.1\cdot m \qquad b := 0.3\cdot m \qquad w := 82\cdot kg\cdot g \qquad w = 804.15N$

$T := \dfrac{w\cdot a\cdot \cos(35\cdot deg)}{(a+b)\cdot \cos(10\cdot deg)} \qquad T = 525.5N$

(b) The normal (horizontal) force and the force of static friction (vertical force) are

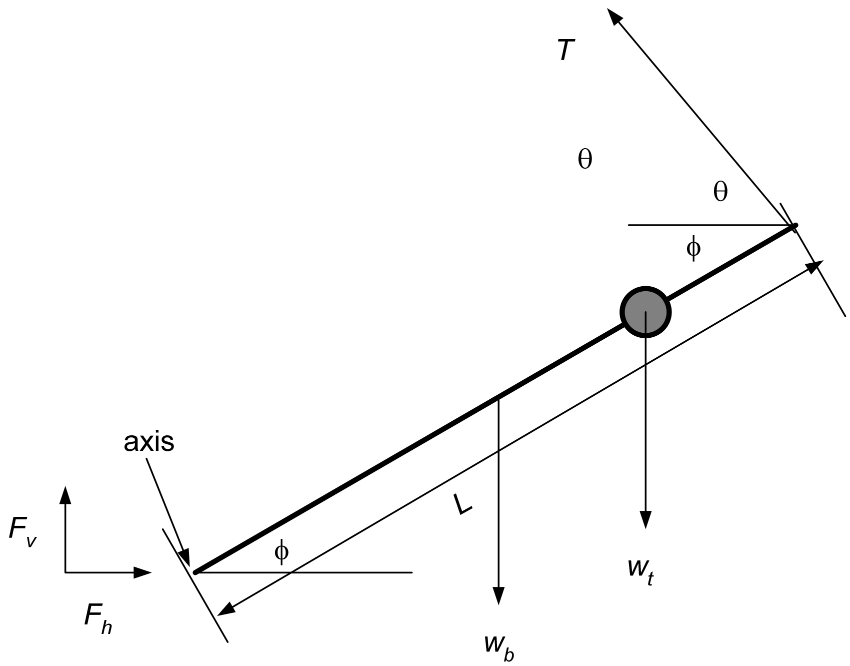
$n := T\cdot \sin(25\cdot deg) \qquad n = 222.1N$

$f_s := w - T\cdot \cos(25\cdot deg) \qquad f_s = 327.8N$

(c) Since we know both the force of static friction and the normal force, we can calculate the coefficient of static friction:

$\mu_s := \dfrac{f_s}{n} \qquad \mu_s = 1.48$

Problem 11.52



The force diagram is as shown.

Note that $\phi = 30^\circ$ and $\phi + \theta = 70^\circ$, so $\theta = 40^\circ$.

We will need all of these angles.

We write the vertical and horizontal force equations and the torque equation:

horizontal

$$F_h - T\cos\theta = 0$$

vertical

$$F_v + T\sin\theta - w_b - w_t = 0$$

torque

$$\left(T\sin 70^\circ\right)L - w_b g\left(\frac{L}{2}\cos\phi\right) - w_t\left(\frac{3}{4}L\cos\phi\right) = 0$$

Note that $T\sin 70^\circ$ is the component of T perpendicular to the bridge; the torque is that component times the perpendicular moment arm (length of bridge). Note that L cancels.

(a) The tension is

$\phi := 30\cdot\text{deg}$ $\theta := 40\cdot\text{deg}$ $L := 40\cdot\text{m}$

$w_t := 3\cdot 10^4\cdot\text{kg}\cdot g$ $w_t = 2.94\times 10^5\ \text{N}$ $w_b := 1.2\cdot 10^4\cdot\text{kg}\cdot g$ $w_b = 1.18\times 10^5\ \text{N}$

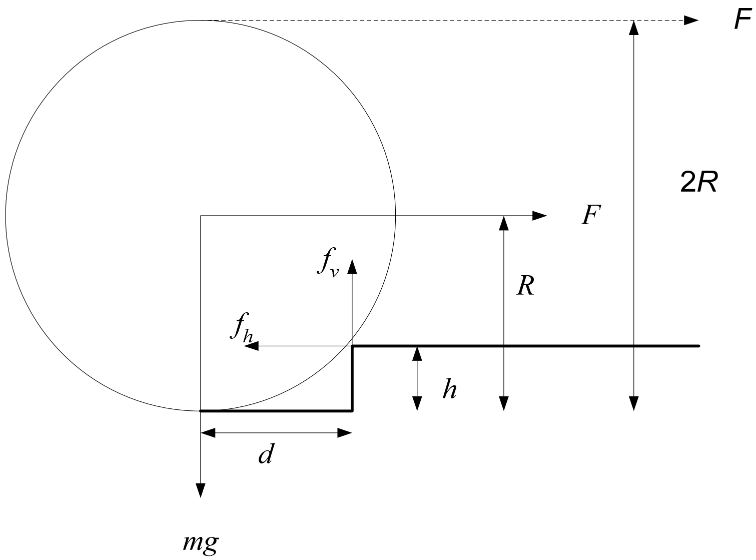
$$T := \frac{w_t\cdot\frac{3}{4}\cdot\cos(\phi) + w_b\cdot\left(\frac{1}{2}\cdot\cos(\phi)\right)}{\sin(70\cdot\text{deg})} \qquad T = 2.58\times 10^5\ \text{N}$$

(b) The vertical and horizontal components of the force at the hinge are

$$F_v := w_b + w_t - T\cdot\sin(\theta) \qquad F_v = 2.46\times 10^5\ \text{N}$$

$$F_h := T\cdot\cos(\theta) \qquad F_h = 1.97\times 10^5\ \text{N}$$

Extra Credit Problem 11.72



A careful force diagram is important!

We assume that the wheel is just barely off the ground, so that we can ignore any normal force on the bottom of the wheel.

It will turn out that we will need only the torque equation. In this problem, we will calculate torques by multiplying forces by perpendicular moment arms.

(a) We calculate the torque around the corner of the step. In this way, the torques exerted by the corner of the step are zero. The torque equation is therefore

$$-F\left(R-h\right)+mgd=0$$

A careful inspection of the diagram will show that

$$d=\sqrt{R^2-\left(R-h\right)^2}=\sqrt{2Rh-h^2}$$

Be sure you work out the geometry carefully, so that you understand this equation. Solving for F and substituting for d , we obtain

$$F=\frac{mgd}{R-h}=\frac{mg\sqrt{2Rh-h^2}}{R-h}$$

(b) Now we assume the force acts at the top of the wheel. The torque equation is

$$-F\left(2R-h\right)+mgd=0 \quad \text{or}$$

$$F=\frac{mgd}{2R-h}$$

and substituting for d as before,

$$F=\frac{mg\sqrt{2Rh-h^2}}{2R-h}$$

This force is smaller than the force we found for part (a)—not surprising, since the moment arm is greater.

Extra Credit Question Q11.7

It should be evident that the person's center of mass must be directly over the point of support—the toes, if standing on tip-toe. Otherwise, there would be a torque—the person's weight times the perpendicular moment arm measured from the toes. That torque will push the feet down, until the center of mass is directly over the point(s) of support.

When someone stands on tip-toe, s/he leans forward a little so that the center of mass is directly over the toes, so that no torque is exerted. But if the person is standing against a wall, s/he cannot lean forward, and hence cannot stand on tip-toe!

Note that if the person's toes are up against a cabinet that is not too high, it is possible to lean forward and balance on tip-toe.