Problems
Class 24
Chapter 9

Question Q9.18

The conservation of energy equation for this problem is

$$\Delta KE_{mass} + \Delta KE_{pulley} + \Delta PE_{mass} = 0$$

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 - mgh = 0$$

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

As the moment of inertia and the rotational kinetic energy increase, the kinetic energy and speed of the descending mass must decrease, since the sum of the two kinetic energies must always add up to *mgh*.

See also the more detailed explanation in 9.48 below.

Exercise 9.34

Please see Fig. 9.29 on page 310. Let the side of the square be *L* = 0.4 m, and the mass be *m* = 0.2 kg.

(a) The distance of each mass from the axis at *O* is $\sqrt{\left(\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^2} = \sqrt{\frac{L^2}{2}} = \frac{L}{\sqrt{2}}$. Hence the moment of inertia about this axis is

$$I = 4m\left(\frac{L}{\sqrt{2}}\right)^2 = 2mL^2 = 0.064 \text{ kg-m}^2$$

(b) The axis is the line AB. The perpendicular distance of each mass from the axis of rotation is *L*/2. Hence

$$I = 4m\left(\frac{L}{2}\right)^2 = mL^2 = 0.032 \text{ kg-m}^2$$

(c) Now the axis is a diagonal of the square. Two of the masses lie on the diagonal, and so make no contribution. The other two are at a distance $L/\sqrt{2}$ from the axis (see part (a)). So the moment of inertia is

$$I = 2m\left(\frac{L}{\sqrt{2}}\right)^2 = mL^2 = 0.032 \text{ kg-m}^2$$

Exercise 9.36

We have three point masses of mass *m*, one at the center of a massless rod of length *L*, and the third at the center.

(a) There are two point masses at the end of the rod, each a distance *L/2* from the axis; the third mass, at the center, does not contribute; hence the moment of inertia is

$$I = 2m\left(\frac{L}{2}\right)^2 = \frac{1}{2}mL^2$$

(b) Now the axis is a distance *L/4* from one end. So two of the masses are a distance $L/4$ from the axis, and the third is a distance $3L/4$. Hence the moment of inertia is

$$I = 2m\left(\frac{L}{4}\right)^2 + m\left(\frac{3L}{4}\right)^2 = \frac{2mL^2}{16} + \frac{9mL^2}{16}$$
$$= \frac{11}{16}mL^2$$

Exercise 9.48

This problem is similar to Example 9.9, which we worked in class. We use conservation of energy to find the speed of the descending mass after it has fallen a distance *h*:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

But the speed of the descending mass is the same as the speed at a point on the pulley's rim: $v = R\omega$ for both. BE SURE THAT YOU UNDERSTAND THIS POINT.

(a) The energy equation becomes

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\frac{I}{R^2}v^2$$

$$mgh = \frac{v^2}{2}\left(m + \frac{I}{R^2}\right)$$

We solve for the speed, and then substitute $I = MR^2$, the value for a cylindrical shell (see page 299).

$$v = \sqrt{\frac{2mgh}{\left(m + \frac{I}{R^2}\right)}} = \sqrt{\frac{2mgh}{\left(m + \frac{MR^2}{R^2}\right)}} = \sqrt{\frac{2mgh}{m+M}}$$

$$= \sqrt{\frac{2gh}{1 + \frac{M}{m}}}$$

In the last step, we have divided both numerator and denominator by *m*.

(b) Compare this result with the result of Example 9, page 301. It should take only a moment to persuade yourself that the denominator in this problem is larger, and so the speed of the descending mass is smaller. Make up some numbers and check if you're not sure.

But the same potential energy is divided between the kinetic energy of the mass and that of the pulley. If the latter is bigger, the former must be smaller. In equation form, consider the our energy equation in the form:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\frac{I}{R^2}v^2$$

Since *mgh* stays the same, if the moment of inertia increases, then the speed of the descending mass **must** decrease.

Exercise 9.49

(a) We can use the analysis of 9.48:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

We know that the kinetic energy of the disk is 4.5 J. Since the pulley is a solid cylinder (disk), we know $I = \frac{1}{2}MR^2$. We have

$$KE_{pulley} = \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\frac{v}{R}\right)^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{v^2}{R^2}$$

$$KE_{pulley} = \frac{1}{4}Mv^2 \quad \text{or}$$

$$v = \sqrt{\frac{4KE_{pulley}}{M}}$$

This speed (which, recall, is the speed of the descending mass) is

$$KE_{pulley} := 4.5\cdot J \qquad M := 2.5\cdot kg$$

$$v := \sqrt{\frac{4\cdot KE_{pulley}}{M}} \qquad v = 2.683\frac{m}{s}$$

The kinetic energy of the descending stone is therefore

$$m_s := 1.5\cdot kg$$

$$KE_s := \frac{1}{2}\cdot m_s\cdot v^2 \qquad KE_s = 5.4J$$

We can now set the change in potential energy of the mass equal to the total change in kinetic energy (KE of stone + KE of disk), and solve for the distance that the stone must fall:

$$\Delta PE + \Delta KE = 0$$

$$-m_s gh + KE_{stone} + KE_{disk} = 0$$

$$h = \frac{KE_{stone} + KE_{disk}}{m_s g}$$

and substituting numbers,

$$h := \frac{KE_s + KE_{pulley}}{m_s\cdot g} \qquad h = 0.673m$$

(b) The pulley's fraction of the kinetic energy is

$$\frac{KE_{pulley}}{\left(KE_s + KE_{pulley}\right)} = 0.455 \quad \text{or } 45.5\%$$

Problem 9.92

We again use conservation of energy, in the form

$$\Delta PE + \Delta KE = 0\,.$$

The box loses potential energy, and the box, the pulley, and the cylinder all gain kinetic energy. We have

$$\Delta PE + \Delta KE = 0$$

$$-m_B gh + \frac{1}{2} m_B v^2 + \frac{1}{2} I_{cyl} \omega_{cyl}^2 + \frac{1}{2} I_p \omega_p^2 = 0$$

We must substitute separately for ω_{cyl} and ω_p. To see why, think about the speed of a knot in the string. That knot, and the descending mass, must both be moving at the same speed v. Hence we have

$$v = R_{cyl}\omega_{cyl} = R_p \omega_p \quad \text{or}$$

$$\omega_{cyl} = \frac{v}{R_{cyl}} \quad \text{and} \quad \omega_p = \frac{v}{R_p}$$

We substitute these results in the energy equation:

$$-m_B gh + \frac{1}{2} m_B v^2 + \frac{1}{2} I_{cyl} \omega_{cyl}^2 + \frac{1}{2} I_p \omega_p^2 = 0$$

$$-m_B gh + \frac{1}{2} m_B v^2 + \frac{1}{2} I_{cyl} \left(\frac{v}{R_{cyl}}\right)^2 + \frac{1}{2} I_p \left(\frac{v}{R_p}\right)^2 = 0$$

$$-m_B gh + \frac{1}{2} m_B v^2 + \frac{1}{2}\left(\frac{1}{2} M_{cyl} R_{cyl}^2\right)\frac{v^2}{R_{cyl}^2} + \frac{1}{2}\left(\frac{1}{2} M_p R_p^2\right)\frac{v^2}{R_p^2} = 0$$

where I have substituted for I_{cyl} and I_p (see page 299). Next, we factor out the speed and collect terms:

$$m_B gh = \frac{v^2}{2}\left\{ m_B + \frac{1}{2} M_{cyl} + \frac{1}{2} M_p \right\}$$

Finally, we solve for the speed and substitute numbers:

$h := 1.5 \cdot m \qquad m_B := 3 \cdot kg \qquad M_{cyl} := 5 \cdot kg \qquad M_p := 2 \cdot kg$

$$v := \sqrt{\frac{2 \cdot m_B \cdot g \cdot h}{m_B + \frac{1}{2} \cdot M_{cyl} + \frac{1}{2} \cdot M_p}} \qquad v = 3.68 \frac{m}{s}$$

Note that we did not need either radius.

Extra Credit Problem 9.70

In this problem, we do not know either the radius or the moment of inertia of the drum.

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = KE_m + KE_{drum}$$

We can find the kinetic energy and speed of the descending mass as follows:

$$KE_m = \frac{1}{2}mv^2 = mgh - KE_{drum} = mgh - 250\ \text{J}$$

$$v = \sqrt{\frac{2\left(mgh - KE_{drum}\right)}{m}}$$

$$KE_{drum} := 250 \cdot J \qquad KE_m := m_d \cdot g \cdot h - KE_{drum}$$

$$v_d := \sqrt{\frac{2 \cdot KE_m}{m_d}} \qquad v_d = 8.05 \frac{m}{s}$$

(a) Now consider Mars: We saw in Exercise 9.49 that the geometry (moment of inertia) of the rotating mass (here, the drum) determines the ratio of the kinetic energies of the drum and the descending mass. *That ratio must be the same on earth as on Mars.*

To put it another way, as the string unwinds, $v = R\omega_{drum}$ will be the same on both planets.

Hence the ratio of $\frac{1}{2}mv^2$ to $\frac{1}{2}I_{drum}\omega^2 = \frac{1}{2}I_{drum}\frac{v^2}{R_{drum}^2}$ must be the same on both.

Hence, the change in the potential energy of the descending mass must be *the same* on Mars as on the earth. In other words,

$$mgh = mg_{Mars}h_{Mars}$$

or

$$h := 5 \cdot m \qquad g_{mars} := 3.71 \cdot \frac{m}{sec^2}$$

$$h_{mars} := \frac{g \cdot h}{g_{mars}} \qquad h_{mars} = 13.2m$$

(b) The speed of the mass on mars will be the same as on Earth: 8.05 m/sec.