PROBLEMS
CLASS 2
YF Chapter 2

Question: On a planet where the value of g is one-half the value on Earth, an object is dropped from rest and falls to the ground. How is the time needed for it to reach the ground from rest related to the time required to fall the same distance on Earth?

On both the earth and the planet, the object falls the same distance, under constant but numerically different accelerations. Hence the distance fallen can be written

$$x = 1/2 a_e t_e^{\,2} = 1/2 a_p t_p^{\,2}$$

where the subscripts *e* and *p* represent the earth and the planet, respectively. Apparently, then,

$$a_e t_e^{\,2} = a_p t_p^{\,2}$$

$$\frac{t_p^{\,2}}{t_e^{\,2}} = \frac{a_e}{a_p} = 2$$

since the gravitiational acceleration of the earth is twice that of the planet. Hence the time of fall on the planet is given by

$$t_p^{\,2} = 2t_e^{\,2}, \text{ or}$$

$$t_p = \sqrt{2}t_e \simeq 1.4t_e$$

Question 2.17

One approach is to note that (taking positive down), the first rock is described by $x_1 = \frac{1}{2}gt^2$ and the second, dropped one second later, by $x_2 = \frac{1}{2}g\left(t-1\right)^2$. We make up a table:

time	x_1	x_2	x_1 - x_2
1	0.5 g	0	0.5 g
2	2g	0.5 g	1.5 g
3	4.5 g	2 g	2.5 g
4	8 g	4.5 g	3.5 g

It is evident from the table that the spacing between the drops increases. Figure 2.22 (page 53) may give you some intuitive insight into this result. We could also calculate $x_1 - x_2$ directly:

$$\begin{aligned}\text{spacing} = x_1 - x_2 &= \frac{1}{2}gt^2 - \frac{1}{2}g\left(t-1\right)^2 \\ &= \frac{1}{2}gt^2 - \frac{1}{2}g\left(t^2 - 2t + 1\right) \\ &= g\left(t - \tfrac{1}{2}\right)\end{aligned}$$

Clearly the spacing increases with time, and the spacings calculated from the table and the equation agree. Units are seconds and meters.

Exercise 2.14

The average acceleration is defined as $a_{av} = \dfrac{v(t_2) - v(t_1)}{t_2 - t_1}$. The instantaneous acceleration $a_x(t)$ is the slope of a tangent to the v_x versus *t* graph at any instant *t*
.

We need one unit conversion: $60\ \text{km/h} = 16.7\ \text{m/s}$

(a) (i) $a_{\text{av-}x} = \dfrac{16.7\ \text{m/s} - 0}{10\ \text{s}} = 1.7\ \text{m/s}^2$.

(ii) $a_{\text{av-}x} = \dfrac{0 - 16.7\ \text{m/s}}{10\ \text{s}} = -1.7\ \text{m/s}^2$.

(iii) $\Delta v_x = 0$ and $a_{\text{av-}x} = 0$.

(iv) $\Delta v_x = 0$ and $a_{\text{av-}x} = 0$.

(b) At $t = 20\ \text{s}$, v_x is constant and $a_x = 0$. At $t = 35\ \text{s}$, the graph of v_x versus *t* is a straight line and so the average acceleration is equal to the instantaneous acceleration; but we have already calculated the former, in (a), part (ii): $a_x = a_{\text{av-}x} = -1.7\ \text{m/s}^2$.

Exercise 2.31

(a) The instantaneous acceleration a_x at time *t* is the slope of the tangent to the v_x versus *t* curve at time *t*.

At $t = 3\ \text{s}$, the v_x versus *t* curve is a horizontal straight line, with zero slope. Thus $a_x = 0$.

At $t = 7\ \text{s}$, the v_x versus *t* curve is a straight-line segment with slope

$$\frac{45\ \text{m/s} - 20\ \text{m/s}}{9\ \text{s} - 5\ \text{s}} = 6.3\ \text{m/s}^2. \text{ Thus } a_x = 6.3\ \text{m/s}^2.$$

At $t = 11\ \text{s}$ the curve is again a straight-line segment, now with slope

$$\frac{-0 - 45\ \text{m/s}}{13\ \text{s} - 9\ \text{s}} = -11.2\ \text{m/s}^2. \text{ Thus } a_x = -11.2\ \text{m/s}^2.$$

(b) We can use the constant acceleration equations only for time intervals during which the acceleration is constant. **Strategy:** break the motion up into constant acceleration segments and apply the constant acceleration equations for each segment.

For the time interval $t = 0$ to $t = 5\ \text{s}$ the acceleration is constant and equal to zero.
For the time interval $t = 5\ \text{s}$ to $t = 9\ \text{s}$ the acceleration is constant and equal to $6.25\ \text{m/s}^2$.
For the interval $t = 9\ \text{s}$ to $t = 13\ \text{s}$ the acceleration is constant and equal to $-11.2\ \text{m/s}^2$.
During the first 5 seconds the acceleration is constant and equal to zero
$v_{0x} = 20\ \text{m/s}$; $a_x = 0$; and hence

$$x - x_0 = (20\ \text{m/s})(5\ \text{s}) = 100\ \text{m};$$ distance traveled in the first 5 seconds.

During the interval $t = 5\ \mathrm{s}$ **to 9 s** the acceleration = 6.3 m/s^2. It is convenient to restart our clock so the interval starts at time $t = 0$ and ends at time t = 4 s.

$v_{0x} = 20\ \mathrm{m/s}$, $a_x = 6.25\ \mathrm{m/s^2}$, $t = 4\ \mathrm{s}$

$$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$$

$$x - x_0 = (20\ \mathrm{m/s})(4\ \mathrm{s}) + \tfrac{1}{2}(6.25\ \mathrm{m/s^2})(4\ \mathrm{s})^2 = 80\ \mathrm{m} + 50\ \mathrm{m} = 130\ \mathrm{m}.$$

At $t = 9\ \mathrm{s}$ the officer is at $x = 230\ \mathrm{m}$, so she has traveled 230 m in the first 9 seconds.

During the interval $t = 9\ \mathrm{s}$ **to** $t = 13\ \mathrm{s}$ $a_x = -11.2\ \mathrm{m/s^2}$. Again we reset our clock so this interval begins at time $t = 0$ and ends at time $t = 4\ \mathrm{s}$.

$v_{0x} = 45\ \mathrm{m/s}$; $t = 4\ \mathrm{s}$

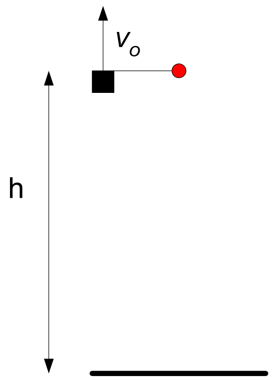
$$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$$

$$x - x_0 = (45\ \mathrm{m/s})(4\ \mathrm{s}) + \tfrac{1}{2}(-11.2\ \mathrm{m/s^2})(4\ \mathrm{s})^2 = 180\ \mathrm{m} - 89.6\ \mathrm{m} = 90.4\ \mathrm{m}.$$

Thus at t = 13 seconds, the total distance traveled is

$$100\ \mathrm{m} + 130\ \mathrm{m} + 90.4\ \mathrm{m} = 320\ \mathrm{m}$$

Exercise 2.44



The ball is released when the balloon is h = 40 m above the ground. The balloon is moving upward at $v_0 = 40\ \mathrm{m/s}$, and hence the ball, as it is released, has the same upward initial velocity. We choose positive up, and the ground level as x = 0. Our equations for the stone are

$$v = v_0 - gt$$

$$x = x_0 + v_0 t - \frac{1}{2} g t^2$$

$$v^2 = v_0^2 - 2g\left(x - x_0\right)$$

Note, of course, that $x_0 = h$.

(a)

$h := 40 \cdot \mathrm{m}$ $\qquad v_0 := 5.0 \cdot \frac{\mathrm{m}}{\mathrm{sec}}$

$t := 0.25 \cdot \mathrm{sec}$ $\qquad v := v_0 - g \cdot t$ $\qquad v = 2.55\mathrm{m}\,\mathrm{sec}^{-1}$

$x := h + v_0 \cdot t - \frac{1}{2} \cdot g \cdot t^2$ $\qquad x = 40.9\mathrm{m}$

$t := 1.0 \cdot \mathrm{sec}$ $\qquad v := v_0 - g \cdot t$ $\qquad v = -4.81\mathrm{m}\,\mathrm{sec}^{-1}$

$x := h + v_0 \cdot t - \frac{1}{2} \cdot g \cdot t^2$ $\qquad x = 40.1\mathrm{m}$

Note the signs of the velocity; in the first case, the ball is moving up; in the second, down.

(b) The ball strikes the ground when x = 0; hence we have

$0 = -4.9t^2 + 5.0t + 40$; and using the quadratic formula

$a := -4.9 \cdot \frac{\mathrm{m}}{\mathrm{sec}^2}$ $\qquad b := 5 \cdot \frac{\mathrm{m}}{\mathrm{sec}}$ $\qquad c := 40 \cdot \mathrm{m}$

$t_1 := \frac{-b + \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$ $\qquad t_1 = -2.39\mathrm{sec}$

$t_2 := \frac{-b - \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$ $\qquad t_2 = 3.41\mathrm{sec}$

We want the second result, of course. How do we interpret the first, negative time?

(c) The ball strikes the ground at a speed

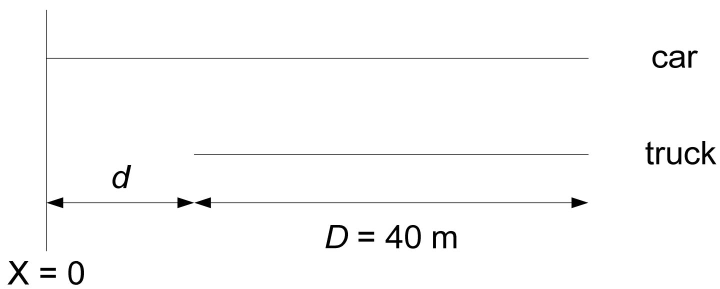
$v := v_0 - g \cdot t_2$ $\qquad v = -28.5\mathrm{m}\,\mathrm{sec}^{-1}$

(d) The velocity of the ball is *v* = 0 at the highest point in the path. We could first find the time at which *v* = 0 and then use that time to find the maximum height. An alternative approach is to solve the third equation for x. Again, we set *v* = 0.

$$v^2 = v_0^2 - 2g\left(x - h\right) \quad \text{recall } x_0 = h$$
$$0 - v_0^2 - 2gh = -2gx$$
$$x = \frac{v_0^2 + 2gh}{2g}$$

$$x := \frac{v_0^{\,2} + 2\cdot g\cdot h}{2\cdot g} \qquad x = 41.3\text{m}$$

Problem 2.69



Both the car and the truck start from rest, so the equations describing their motion are

$$x_{car} = \frac{1}{2}a_{car}t^2 \qquad x_{tr} = d + \frac{1}{2}a_{tr}t^2$$
$$v_{car} = a_{car}t \qquad v_{tr} = a_{tr}t$$

(a) The car overtakes the truck when $x_{car} = x_{truck}$. But if we use this equation directly, we would have one equation in two unknowns, *d* and *t*. Instead, we note that the car catches up with the truck when the car has gone a distance *d* + *D*:

$$x_{car} = x_{truck}$$
$$d + D = d + \frac{1}{2}a_{tr}t^2$$
$$t = \sqrt{\frac{2D}{a_{tr}}}$$

Next, we do the arithmetic:

$$D := 40\cdot\text{m} \qquad a_{tr} := 2.10\cdot\frac{\text{m}}{\text{sec}^2}$$

$$t := \sqrt{\frac{2\cdot D}{a_{tr}}} \qquad t = 6.17\text{sec}$$

(b) We calculate how far the car has moved, from which we can calculate *d*:

$$a_{car} := 3.40 \cdot \frac{m}{sec^2}$$

$$x_{car} := \frac{1}{2} \cdot a_{car} \cdot t^2 \qquad x_{car} = 64.76m$$

$$h := x_{car} - D \qquad h = 24.76m$$

(c)

$$v_{car} := a_{car} \cdot t \qquad v_{car} = 21 m sec^{-1}$$

$$v_{tr} := a_{tr} \cdot t \qquad v_{tr} = 13 m sec^{-1}$$

(d)

Problem 2.76

Please refer to the figure on page 69. I will choose $x = 0$ to be the height of the person's head—hence for the egg at the top of the building, $x_0 = 44.2$ m. I will also choose positive up. Our equations are

$$x = x_0 - \frac{1}{2} g t^2$$
$$v = -gt$$
$$v^2 = v_0^2 - 2g(x - x_0)$$

The egg reaches the top of the person's head when $x = 0$; so we set $x = 0$ in the first equation and solve:

$$x_0 := 44.2 \cdot m \qquad t := \sqrt{\frac{2 \cdot x_0}{g}} \qquad t = 3.00 sec$$

In that time, the person moving at $V := 1.2 \cdot \frac{m}{sec}$ moves a distance

$$X := V \cdot t \qquad X = 3.6m$$

Extra Credit: Question 2.20

Let's write the equations for the two balls. I will choose *x* = 0 at the bottom of the building, and positive up. Hence our equations are

$$\begin{array}{ll} \text{dropped from top} & \text{thrown up from bottom} \\ x_1 = h - \frac{1}{2}gt^2 & x_2 = v_{20}t - \frac{1}{2}gt^2 \\ v_1 = -gt & v_2 = v_{20} - gt \\ v_1^2 = -2g\left(x_1 - h\right) & v_2^2 = v_{20}^2 - 2gx_2 \end{array}$$

Be sure you understand where these equations come from. Note that the velocity of the second ball is 0 at $x_2 = h$ (highest point of path); hence, $v_{20}^2 = 2gh$, and we have for the two balls, using the third equations,

$$\begin{array}{ll} \text{dropped from top} & \text{thrown up from bottom} \\ v_1^2 = -2g\left(x_1 - h\right) \quad \text{or} & v_2^2 = 2gh - 2gx_2 \quad \text{or} \\ v_1^2 - 2gh = -2gx_1 & v_2^2 - 2gh = -2gx_2 \end{array}$$

The balls pass when $x_1 = x_2$; and at $x_1 = x_2$, the equations immediately above give $v_1^2 = v_0^2$; the first velocity is negative and the second positive, since they move in different directions.

The second part is trickier. We start by finding the time at which the two speeds are equal (that is, when the balls are at the same height—see above).

$$\begin{aligned} &\left|v_1\right| = \left|v_2\right| \quad \text{at time } t \\ &gt = v_{20} - gt \qquad \text{but recall } v_{20} = \sqrt{2gh} \\ &gt = \sqrt{2gh} - gt \\ &t = \frac{\sqrt{2gh}}{2g} = \sqrt{\frac{h}{2g}} \end{aligned}$$

The vertical lines in the top equation means magnitude—remember the speeds are equal, not the velocities. Now substitute this value of time into the equation for either x_1 or x_2:

$$\begin{aligned} &x_1 = h - \frac{1}{2}gt^2 = h - \frac{1}{2}g\left(\frac{h}{2g}\right) = h - \frac{1}{4}h \\ &x_1 = \frac{3}{4}h \end{aligned}$$

The balls meet at the same speed at ¾ *h* from the ground. Substituting in the equation for x_2 gives the same result.

Exercise: Show that this time is equal to half the time it takes one ball to fall to the ground, and the other to rise to the top of the building. Hint: Notice that since the acceleration is constant, the *speed* of the dropped ball increases linearly with time, and the *speed* of the thrown ball decreases linearly with time. Therefore, if *T* is the time it takes one ball to fall to the ground and the other to rise to the top, then the two *speeds* (not velocities!) must be equal at time $t = T/2$.